100 aGPSS Models: Cases and Exercises

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Below are 100 aGPSS models. Most of them are meant to be exercises for students, but many should also be seen as cases, i.e. examples of applications of aGPSS.

The first 57 models in this booklet are meant to be studied alongside with the study of the 50 different lessons in the aGPSS textbook. Most of the remaining models are more advanced and often require that all 50 lessons have been studied. None of the models are from the aGPSS textbook, but are all new.

Since there is generally only one exercise in each of the 50 lessons in this textbook, many students and teachers have declared a need for more exercises. One of the aims of this booklet is therefore to answer to this demand. After each exercise with a number lower than 58, there is within parentheses a number that refers to the latest lesson in the textbook that contains the information that is necessary to solve the exercise.

Another purpose of this booklet is to show that GPSS is really a general purpose system with at very wide area of applications. A third purpose to give ideas for project work, which is often the final part of a basic simulation course. Several of the fairly small model in this booklet have later been expanded into a larger model representing an actual situation in a company. In fact, only a few of the models here are quite large with around 200 blocks. We have found it better to present many smaller models than just a few very large ones.

Among the issues studied, we have the establishment of optimal equipment life (model 68), overbooking and price differentiation in the airline industry (66), bidding on stocks of an oil company with uncertainty about the success of an oil exploration process (69), a basic example of the value of perfect information in decisions under risk (92) and a model of stocking of perishable goods in a supermarket (88). We also illustrate some basic ideas of continuous simulation as applied to business problems with the aim of doing market forecasts (73) and evaluation of the value of a high tech corporation (99). We also deal with the evaluation of European options, which for the case of constant volatility can be solved analytically, but for the case of volatility varying over time requires simulation (97). We also discuss simulation based costing (65) and the use of laptop based simulation models for sales support (98).

We present many manufacturing examples (e.g. 63, 67, 75, 82, 83), but also examples of logistics with trucks (100), shipping (77, 78) and taxi operations (72). Some examples are from services like restaurants (94), gas stations (62) and libraries (74). Among other examples we have an IT-exhibition (81), systems with elevators (79, 80), administration of secretarial work (86), software development (93), car maintenance (95), super market retail operations (54, 59, 89) and partner selection (71). There are also examples of oligopoly theory models (85, 90) and of coordination of production, inventory and pricing policy to improve the company's cash flow. (87, 91).

It should finally be mentioned that several exercises are built on programs in other books. The acknowledgements to these exercises are given at the top of the suggested solutions, which are available in electronic form (see www.aGPSS.com). It should here be mentioned that the many of these examples are case studies from T. Schriber's famous "red book", *Simulation Using GPSS.* Y. Matsuyama's careful review of the present booklet, leading to many substantial improvements, must also be acknowledged.

1. You have started to work on the assembly line at a car factory to make some extra money. You do riveting on the distributor box of the newest model. You do piece-work, but you find it annoying that you cannot determine your own production speed, since you are dependent on the guy who works just in front of you on the line and who mounts the boxes. The boxes arrive at his station with a time distance of between 5 and 9 seconds. It then takes him between 6 and 12 seconds to mount a box. It then takes between 3 and 5 seconds for you to do the riveting. (All times are uniformly distributed.) What percent of the time is each one of you two busy working, seen over a period of 10 minutes? (5)

2. Combine Models 08 and 13 so that you from the same program get statistics on both waiting time and time spent in the store. (8)

3. Change Model11, in the first instance so that customers come on average every 28 (instead of 18) minutes apart. Next change also the half-width of the ADVANCE block from 5 minutes to 25 minutes, implying that service time varies between 0 and 50 minutes. Compare the table of the first run with the table in Figure 47. (8)

4. Two types of customer arrive at a one-chair barber shop. Customers of the first type want only a haircut. Their InterArrival-Time distribution is 35 $\pm$ 10 minutes. Customers of the second type want a shave as well as a haircut. Their InterArrival-Time distribution is 60 $\pm$ 20 minutes. The barber provides service to his customers on a first-come, first-served basis. It takes the barber 18 $\pm$ 6 minutes to give a haircut. When he gives a shave, 10 $\pm$ 2 minutes are required. Collect data on the waiting line that forms ahead of the barber, both total and for the two types of customers. Run the model through eight hours of simulated time. (9)

5. A port has two jetties, each of which can be used for unloading by one boat at a time. Boats arrive at the port periodically and must wait if no jetty is currently free. When a jetty is available, a boat may dock and start to unload. When this activity has been completed, the boat leaves the jetty and sails away. The port authority has a pool of three tugs. A tug is required for docking and when a boat leaves its jetty. On average boats arrive 10 hours apart, but this can vary between 7 and 13 hours. It takes 14 $\pm$ 6 hours to unload. Docking and leaving, using tugs, take 2 hours each. Simulate for 28 full days of 24 hours. (11)

6. One doctor and two nurses work in a clinic. Patients arrive every 15 $\pm$ 10 minutes. Patients first go to one of the nurses (whichever one is free) for tests, which takes 25 $\pm$ 10 minutes. They then proceed to the doctor for examination, which takes 15 $\pm$ 10 minutes. Since some patients have complained about very long waiting times, the doctor is considering a different arrangement. He realizes that if the nurse, who has only made the tests on the patient, also stayed with the patient during the examination, the time for examination would decrease to 10 $\pm$ 9 minutes. Write a program for both the present and the proposed alternative. Simulate for a seven-hour day to see if there will be fewer patients, who have very long waiting times. (11)

7. Alter Model19 to allow for two lathes and two grinders. The average processing times for the machines are now doubled, but the half-widths are unchanged. (11)

8. Modify Model19, so that instead of starting production of a unit of A every 25 minutes and one of B every 50 minutes, production of a unit of A or B commences as soon as the lathe is free.

Compare the results of this new program with those of the old program. (12)

9. A student owns two cars. Each day, he uses a car to get to school and back. Car 1 is old and has probability 0.79 of starting on any day, while car 2, even older, starts with probability 0.71. For either car, once it is started, the chance that it completes a trip to or from school is 0.95. Compute the probability that he will make it to school and back on any particular day. First do the calculation manually, then write a GPSS program to do it. (13)

10. Ambulances are dispatched at a rate of one every 10 $\pm$ 7 minutes in a large metropolitan area. Fifteen percent of the calls are false alarms, which require 12 $\pm$ 2 minutes to complete. All other calls can be one of two kinds: The first kind are classified as serious. They constitute 25 percent of the non-false alarm calls and take 25 $\pm$ 5 minutes to complete. The remaining calls take 20 $\pm$ 10 minutes to complete. There are three available ambulances, and they are on call at any time. Simulate the system for 500 calls to be completed. Give statistics on how long serious alarm cases wait before an ambulance is on its way. (13)

11. People arrive at a self-service cafeteria at the rate of one every 40 $\pm$ 20 seconds. Forty percent go to the sandwich counter, where one worker makes a sandwich in 60 $\pm$ 30 seconds. The rest go to the main counter, where one server spoons the prepared meal on to a plate in 45 $\pm$ 30 seconds. Then all customers pay a single cashier, which takes 25 $\pm$ 10 seconds. For all customers, eating takes 20 $\pm$ 10 minutes. After eating, 40 percent of the people return to buy a coffee and pay the cashier for this. Drinking coffee takes 10 $\pm$ 2 minutes.

Simulate a day of five hours. The output should include data on average time spent in the cafeteria, the maximum number of people at any time and average waiting time in front of the cashier. (13)

12. Modify Model26 to take into account that 40 percent of the customers are impatient and balk if Joe is busy, while the remaining 60 percent balk if there are 4 or more persons waiting. (16)

13. We simulate a simple bicycle store for 250 days of 8 hours. The store has space for 15 bikes in stock. The stock is replenished with six bicycles every 10^{th} day. If there is no space in the inventory room, the delivery of the bike has to wait until there is such space. Customers wanting to buy a bike arrive every *x* hours. Test with *x* = 10 and 20 to see if deliveries of new bikes are delayed or if customers have to wait for a bike. (17)

14. In traffic systems, like that of Model31, it is possible, when the light is green for a short time (i.e. there is a red light on the other street), that all the cars that are waiting might not be able to cross the street before the traffic light turns red again. Take this problem into account by modifying Model31 according to the following assumptions: Before crossing the street, each car will be in the position of first car in the waiting line. It will be in this position for 1 - 3 seconds and from there cross the street, if there is not a red light. (18)

15. We model a store, having two salespersons, each needing 4 $\pm$ 2 minutes to serve a customer. To determine the IAT of the customers, one has recorded the arrival times of various customers during the day, leading to the set of numbers presented on the next page, dealing with minutes after the opening of the store.

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0.6, 1.6, 1.8, 3.4, 3.8, 5.2, 7.1, 9.3, 10.1,
12.8, 14, 16, 19.5, 24.9, 25.1, 29.2, 34.8,
35.6, 39.3, 39.8, 40.8, 45.1, 46.2, 46.3, 51.2,
52.4, 55.3, 57, 57.6, 62.3, 63.8, 64.9,70.1,
70.2, 72, 76.5, 77.7, 80.9, 81.6, 82.7, 85, 89.2,
89.4, 90.3, 91.7, 94.2, 95.1, 96.8, 99.4
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On the basis of these numbers one can calculate the time between the arrivals and then distribute them into six bins for 0 - 1, 1 – 2, .. , 5 - 6 minutes etc.

Next one can formulate an empirical IAT function and use this function in the model. Simulate the operations of the store for 10 hours. (20)

16. In a store, sales only refer to one product, but with different prices for two types of customers, arriving 18 ± 6 minutes apart: Members, accounting for 60 percent, pay $ 20, but the rest must pay $ 30. If there are no units in stock, when a customer arrives, the customer waits until new stocks arrive. Every day 25 new units are delivered provided at least one customer is waiting and the store has enough cash to pay $ 15 for each unit. When we start, the company has 400 in cash. Simulate an operation of 30 days of 8 hours. (22)

17. A store in central Stockholm is handled by two salespersons, who serve the customer who has waited the longest time. The InterArrival Times of the customers are exponentially distributed with a mean of 1.5 minutes. The service times are normally distributed with a mean of 8 minutes and a standard deviation of one minute. Customers balk if there are already a total of 6 customers waiting. The store is open from 10 AM to 7 PM. After 7 PM no new customers are allowed into the store. The store is closed when all customers who have entered have been served. (23)

18. At a one-car wash facility, it must be decided how many spaces to provide for cars waiting to use the facility. Cars arrive in a Poisson stream (= according to an exponential distribution) with an average InterArrival Time of 5 minutes. Car-washing time is exponentially distributed with a mean of 4 minutes. Potential customers who find no waiting space available go elsewhere to have their car washed.

Build a model of this simple system, then use the model to observe system behavior for the alternatives of one, two and three waiting spaces. For each configuration, simulate for one eight-hour day of operation. Basing your answer on model output, estimate the fraction of potential customers who actually remain at the car wash to be served. (23)

19. Modify Model08 so that Average time/trans, printed in the queue statistics, is also obtained as a savevalue. Total waiting time is calculated by subtracting CL, i.e. the value of the present clock, from this total time at each entry into the queue and by then adding CL to this time at each exit from the queue. At stop time, the **product** of CL and the number of customers remaining in the queue (in the block prior to SEIZE) is also added to the total time. This total waiting time is then finally divided by the number of entries into the queue. (25)

20. The quality of a pot in Exercise 20 in the textbook, and hence the price to be charged, varies in a random fashion. 30 percent of the pots are of low quality and can be fetch a price of $100; 60 percent are of medium quality, fetching a price of $ 150, while 10 percent are of high quality, fetching a price of $ 200. Have the simulation also calculate the revenues of the production during the day. (25)

21. Modify Model39 as follows: Customer arrivals now follow an Erlang distribution with n = 3 and service times follow the normal distribution. Average times are not changed, but service times are, so that 95 percent of all times lie between 15 and 35 minutes. Run for one and two salespeople. (26)

22. A shop buys and sells antiques. Persons selling antiques arrive at the shop according to the exponential distribution with an average IAT of 2 hours. The price paid for an antique item is on average $ 100, but the prices vary according to a normal distribution with a standard deviation of $ 20. The shop always pays cash. The buyers arrive at the shop also according to the exponential distribution, but with an average of 5 hours. The shop sells antique items for $ 150 on average, but prices vary according to a normal distribution with a standard deviation of $ 25. The customers obtain credit for 4 days, but the actual times until payment follow an Erlang distribution with n = 3 and with an average time of 6 days.

The store is open between 9.00 a.m. and 5.00 p.m. each day, also Sundays. When the store is closed at 5.00, all customers leave. At simulation start the inventories are valued at $ 10,000 and there is $ 50,000 in cash. How much cash will the shop have and what is its inventory worth after a month of 30 days? Every item in the inventory is valued at $ 100, irrespective of the purchase price. (26)

23. Change Model52 in the following way: The initial price is to be read in as $ 33,000, but if cash falls below $50,000 the price shall be increased by x percent, where x is a value read from the keyboard. The price is, however, not allowed to increase above $50,000. Test the effect of different values of x. The aim is to be able to do three runs in a row without getting a negative cash balance. (30)

24. In a one-line, one-server queuing system, arrivals occur in a Poisson pattern (i.e. according to the exponential distribution) with a mean rate of 14 arrivals per hour. Service times follow the exponential distribution, but the mean service time depends on the content of the waiting line ahead of the server, as follows:

Contents of the waiting line (minutes)	Mean service time (minutes)
0	5.5
1 or 2	5.0
3, 4 or 5	4.5
6 or more	4.0

Build a model of the system. If the arrival rate increases by one additional arrival per hour, will the server still be able to handle the flow of customers reasonably well, or will the waiting line become much longer? Run the model for 500 customers! (30)

25. For Model40, produce graphs over the development of cash during the day, for various values of the starting position on cash in the morning. Since cash might become negative, the program must be rewritten, so that cash is not handled as a storage, but as a savevalue, e.g. X$cash, which may assume negative values. It should be possible to change the starting value of cash interactively, i.e. without having to change the program. Note that the graph might be illegible, if more than five days are simulated. (30)

26. Customers arrive at a store every 25 seconds on average, following an exponential distribution. Service time is likewise 25 seconds on average, also following an exponential distribution. A customer buys on average for $10.00. The store owner contemplates whichever of one or two salespersons, each costing $ 100 per day, leads to the higher daily profit. Run 40 times. (31)

27. Modify Model15 so that the cost of waiting for the customers can be calculated and printed, assuming that they value their time at an average of $ 6.00 an hour. (32)

28. Rewrite Exercise 12 in the textbook, so that one can calculate the total costs of lost production, repairers and reserve machines, assuming a repairer costs $13.75 per hour, a reserve machine $30 per day and the hourly cost penalty for having fewer than 50 machines in production is $20 per machine. Allow for a change of the value of both the number of repairers and the number of reserve machines. Run the program for all combinations of values on these two variables from 1 to 3 to study variations in costs. (34)

29. Modify Model21 so that one of the two salespeople (or barbers) only works part time - for three hours a day, starting at noon and stopping at 3.00 p.m. The store opens at 9.00 a.m. and closes at 5.00 p.m. Only statistics on waiting time are required. (36)

30. Messages are created at the rate of one every 30 $\pm$ 10 seconds. They are sent over a communication channel, one at a time. Twenty percent of them require a reply, which returns over the same channel after a delay of 60 $\pm$ 30 seconds. Assume the transmission time for the original message to be 25 $\pm$ 15 seconds and for the reply 15 $\pm$ 5 seconds. Assume also that both messages and replies can be stored for transmission, if necessary. Compare the time it takes to complete 100 messages with replies: (1) when the replies have priority and (2) when the messages have priority. (36)

31. Modify Exercise 36 in the textbook. We assume that all arrivals follow the exponential distribution, that stay and operation times follow the Erlang distribution with n = 3, and that the average IAT is 9 h. for the general patients and 8 h. for the operation patients.

Furthermore, we assume that the operation theater does not close when an operation is in progress, but that no new operation starts, if a total of 5 operations have already been carried out since the operation theater was opened in the morning. We now assume that patients requiring an operation have priority for beds over patients who are there for a short hospital stay without operation. (36)

32. In a factory, a tool crib is manned by a single clerk. The clerk checks out tools to mechanics, who use them to repair failed machines. The IATs and the times to process a tool request (both in seconds) depends on the type of tool, as follows:

Category of tool	Mechanic IAT	Service time
1	420 $\pm$ 360	300 $\pm$ 90
2	360 $\pm$ 240	100 $\pm$ 30

The clerk has been serving the mechanics first-come, first-served, irrespective of their requests. Owing to lost production, it costs $9 per hour when a mechanic waits for service at the tool crib, regardless of the tool to be checked out. Management believes that the average number of waiting mechanics can be reduced, if Category 2 requests are serviced at the tool crib before those in Category 1. Model the tool crib for each of the two queue disciplines, simulating each case for an eight-hour work day. Does 'first-come, first-served, within Priority Class' reduce the average number of mechanics waiting in line? Calculate also the total daily cost of lost production for each priority discipline. (36)

33. An office has two copying machines. Secretaries from the general office arrive to use a machine at the rate of one every 5 $\pm$ 2 minutes. They choose a machine at random, whether it is busy or not, and they stay with their choice.

A secretary from the president's office arrives at intervals of 15 $\pm$ 5 minutes. If a machine is free, it is used. If not, the secretary goes to the head of the line for machine number 1. Assuming that all jobs take 6 $\pm$ 3 minutes, determine the time needed for the completion of 100 jobs of all types. (37)

34. Modify the program of exercise 32 above so that, in a single batch run, 40 different days of operation will be simulated for the tool-crib problem. Arrange the model so that both the ‘no priority’ and ‘priority’ queue disciplines are investigated with the same program. (Note that a savevalue determined initially can be used as priority.) The resulting output will provide two samples, each consisting of 40 different ‘total waiting costs’ estimates for the corresponding disciplines. Let aGPSS compute the mean and standard deviation for each of these two samples. How much better is the priority discipline based on these samples? (37)

35. Customers pass through the following servers: a weighbridge (on the way in), an unloading bay, a washer, a loading bay and the weighbridge (on the way out). Queues q1 to q5 form in front of these five servers. q1 and q5 both form in front of the weighbridge. q5 (customers about to leave) has priority over q1.

The InterArrival Time between successive customers is exponential with 17 minutes as the average. The first customer arrives at time 5. No customers are accepted after time 360, but customers already in the system finish being served. Service times are all uniform between the following limits: for the weighbridge 3 – 7, the washer 5 – 7, the unloading bay 8 – 16 and the loading bay 15 - 20. Investigate the reduction in average customer waiting time obtained by acquiring a second loading bay. (37)

36. At an attraction at an amusement park, with a capacity for 8 children, 50 per cent of the children, who have taken a ride, want to repeat the ride. Those who already have taken the ride have to wait in line after those who have not yet taken the ride. Hence, a child will have a higher priority, when newly arrived, than when having already taken the ride. Model this for the case that the children arrive 10 $\pm$ 2 seconds apart and a ride takes 50 seconds. Simulate for a time of 5 minutes. (37)

37. Modify Model73 so that correct statistics for server utilization are also obtained. (38)

38. Modify Model39 to take into account a varying intensity in customer arrivals during the day and the possibility of hiring a part-timer as the second salesperson. Assume that the store is now open between 9 a.m. and 6 p.m. Average IAT is 25 minutes 9 a.m. - 11 a.m.; 15 minutes 11 a.m. - 1 p.m.; 30 minutes 1 p.m. - 4 p.m. and 10 minutes 4 p.m. - 6 p.m.. The specific arrival times are exponentially distributed. The part-timer is willing to work any four hours in a row. Should the part-timer be hired to work between 11 a.m. and 3 p.m. or between 2 p.m. and 6 p.m., if the objective is to sell as much as possible? Assume that all other factors than those mentioned here are as in Model39. (38)

39. In a foundry, finishing work on castings is done by a machine. Only one worker is needed to operate such a machine. The work consists of a sequence of two processes, Process 1 and Process 2. The steps to do the work are:

1. Perform Process 1.
2. Re-position the casting on the machine.
3. Perform Process 2.

4. a) Unload the finished casting from the machine.
   b) Store the finished casting.
   c) Fetch the next rough casting from the storage area.
   d) Load this rough casting onto the machine.
5. Return to step 1 above.

An overhead crane is needed at each step involving movement of the casting, i.e. steps 2 and 4 above. Whenever a crane is not being used for one of these steps, it is in an idle state. The finishing machine goes repeatedly through a single closed cycle consisting of the four steps listed above. The crane, on the other hand, can go through each of two distinct cycles, depending on whether it is being used for step 2 or step 4. The times (in minutes) required to perform steps 1 and 3 in the finishing process are as follows:

Time for process 1	Cumulative frequency	Time for process 2	Cumulative frequency
< 60	0.00	< 80	0.00
70	0.12	90	0.24
80	0.48	100	0.73
90	0.83	110	1.00
100	1.00		

The handling of the casting in step 2 requires 15 $\pm$ 5 minutes. The sequence described by step 4 takes 30 $\pm$ 5 minutes. Management wants a simulation study of the finishing department. It wants to determine what utilizations would result if the number of machines served by a single crane were three, four, five or six. Secondly, it wants to know what the effect would be if two cranes or three cranes were used. For each condition, which is input interactively, simulate for five 40-hour work weeks. In case of a conflict, step 2 is to be given priority over step 4. (38)

40. Customers come to a store with two salespeople on average every 10 minutes following an exponential distribution. The average service time is 15 minutes. Hardly any customer takes less than five minutes or more than half an hour. It appears that the triangular distribution could be a convenient approximation of service times. Simulate for a day of nine hours. (39)

41. Run Exercise 24 above again for 500 customers, but now with a print-out after every 100th customer. Hint: The decimal fraction of *n*/100 is = 0 for every such customer. Use FN$dec. (39)

42. Modify Model22 in such a way that each worker finishes work for the day and goes home as soon as he has produced 14 pots. (40)

43. A party of 100 people has gone to a football game in four buses, each with a capacity of 25 people. When the game is over, each person returns independently to the bus that brought him to the game. The time taken to reach a bus is normally distributed with mean values of 10, 12, 15 and 18 minutes for the four buses. In each case the standard deviation is 2 minutes. When a bus is full, it leaves, and arrives home after a drive that is normally distributed with a mean of 80 and a standard deviation of 5 minutes. Begin a simulation from the time the game finishes and find the time at which the last bus arrives home. (40)

44. A small grocery store consists of three aisles and a single checkout counter. Shoppers arrive at the store in a Poisson pattern with a mean inter-arrival time of 75 seconds. After arriving, each customer takes a basket and may then go down one or more of the three aisles, selecting items as he proceeds.

The probability of going down any particular aisle, the time required to shop an aisle, and the number of items put into the basket in each aisle are as follows:

Aisle	Probability of going down the aisle	Seconds to go down the aisle	Number of items selected
1	0.75	120 ± 60	3 ± 1
2	0.55	150 ± 30	4 ± 1
3	0.82	120 ± 45	5 ± 1

When shopping is complete, customers queue up first-come, first-served at the checkout counter. Here, each customer puts an additional 2 ± 1 "impulse items" into the basket. The checkout time is 3 seconds per item in the basket. After checking out, the customer leaves the basket and departs. Simulate the store for an 8-hour day. Measure the utilization of the checkout counter and the maximum length here. Determine the maximum number of baskets in use. (41)

45. The IATs for parts needing processing is defined as follows:

InterArrival Time (seconds)	Proportion
10 - 20	0.20
20 - 30	0.30
30 - 40	0.50

There are three standard types of part: A, B and C. The proportion of each part, and the mean and standard deviation of the normally distributed processing times, are as follows:

Part type	Proportion	Mean (seconds)	Standard Deviation (seconds)
A	0.5	30	3
B	0.3	40	4
C	0.2	50	7

Compare the waiting times for parts when one, two or three machines work in parallel, where each machine processes any type of part, one part at a time. Simulate for one hour. (42)

46. Simulate a single-server queuing situation, where customers arrive every 60 seconds. The service times of successive customers are uniform random variables between 10 and 90 seconds. Simulate 50 cycles, where one cycle begins and ends with an idle period for the server. For each cycle, calculate the sum of individual waiting times for all customers served during the cycle and the number of customers served during the cycle. (42)

47. Change Model49 as follows: The orders arriving no longer refer to only one unit. Only 40 percent concern a single unit. 35 percent of the orders are for two units and 25 percent of them are for three units. The accounts due will hence refer to different amounts and must be represented by a parameter, since it is possible that e.g. a claim regarding 3 units and hence for a total of $ 75 K ($75,000) will be paid later than a claim regarding 1 unit and for $ 25 K, even though the order for the 1 unit arrived later. The program shall, just like Model49, produce a quarterly report, but also a graph over the development of cash. Run the program five times to study, if there is any substantial risk of a negative cash balance. (42)

48. Modify Model45 in the following ways: Customers arrive following an exponential distribution, with an average InterArrival Time of 2 minutes at 10 a.m., of 1.5 minutes at 3 p.m. and of 1 minute at closing time 6 p.m. (8 hours after start), i.e., more and more customers arrive as they day goes on. Normally seven people work full-time (eight hours a day) in the store. Each salesperson has a separate waiting line. The store is closed in a ‘proper manner’ as in Model30.

Simulation is to be used for deciding whether to hire a part-timer to work towards the end of the afternoon. The part-timer is paid $ 10 per hour. All other factors are as in Model45. Carry out a simulation with a part-timer, starting at what you would consider to be a suitable time. Make a comparison between two starting times based on the profits from the operations. (43)

49. Customers arrive at a bank in an exponential pattern at a mean rate of 200 per hour. Eight teller windows are open at the bank at all times. A separate waiting line forms ahead of each teller. If a teller is free when a customer enters the bank, the customer immediately goes to that teller. Otherwise, the customer joins whichever waiting line is the shortest. Customers then remain in that line. The various types of business that customers transact fall into five categories. In each category service time is exponentially distributed. The relative frequencies of these categories and the corresponding mean service time requirements, are as follows:

Category of business	Relative frequency	Mean service time (seconds)
1	0.10	45
2	0.19	75
3	0.32	100
4	0.24	150
5	0.15	300

Another bank in the vicinity has introduced a ‘Quick line’ queuing system. In this system, customers entering the bank form a single line. Whenever a teller becomes available, the customer at the head of the line goes to that teller. Build a model, which gathers waiting line information for the bank's operation, both as it currently exists, and for a change to a Quick line system. Run the model for a five-hour day in each case. (43)

50. Orders arrive at a factory at the rate of 20 every hour, following an exponential distribution. There are orders for three types of goods: 40 percent are of type A, 35 of type B and 25 of type C. They each require processing in a special machine. Processing in the A machine takes 6 $\pm$ 4 minutes, in the B machine 9 $\pm$ 3 minutes and in the C machine 12 $\pm$ 5 minutes. Simulate for 50 hours to study to what extent orders are held up at the various machines. (45)

51. Modify Model96, so that it provides data on how many times the service of Joe has been interrupted by other transactions. (46)

52. A computer center substation has two workstations. On arrival, users choose the workstation with the shortest waiting line. Students arrive at a rate of one every 8 $\pm$ 2 minutes. They can be interrupted by professors, who arrive at a rate of one every 12 $\pm$ 2 minutes. There is one systems analyst, who can interrupt anyone, but students are interrupted before professors. The systems analyst spends 6 $\pm$ 4 minutes on the workstation and arrives every 20 $\pm$ 5 minutes. Professors and students spend 4 $\pm$ 2 minutes on the workstation. If a person is interrupted, the person joins the head of the queue and resumes service as soon as possible.

Simulate 50 professor or analyst jobs. Estimate the mean length of the student waiting line and the average waiting time at the two terminals. (46)

53. Motorists wishing to cross the strait between the mainland and a small island have to use a ferry. The ferry is moored on the mainland overnight and starts work promptly each day at 7 a.m. It shuttles to and fro between the mainland and the island until approximately 10 p.m. when the service closes down for the night. The ferry has a capacity limit of six cars.

When the ferry arrives at a quay, the cars on the ferry are driven off, and then any waiting cars (up to the maximum of six) are driven on.

When the ferry is fully loaded, or the quay queue is empty, the ferry leaves that side of the strait and starts another crossing, When the ferry has completed a round trip and deposited any passengers on the mainland, the captain checks the time. If it is 9.45 p.m. or later, he closes down the service for the night.

Both on the mainland and the island, cars arrive on average at a rate of nine per hour, following an exponential distribution. Crossing times follow the normal distribution with a mean of 8 minutes and a standard deviation of 0.5. It takes a car 0.5 minutes to drive on or drive off the ferry.

Run the model for one working day on which there are three cars waiting on the mainland and one on the island at 7.00 a.m. Produce a table of the number of cars on the ferry at each crossing. (47)

54. In a retail store, the daily demand for a given item is normally distributed with a mean and standard deviation of 10 and 2 units, respectively. Whenever the retailer's stock-on-hand drops to or beneath a predetermined point, called the re-order point, he places a stock-replenishment order with his supplier regarding a specific replenishment amount, called the re-order quantity, provided there is no unfilled order.

The replenishment arrives at the retail store 6 to 10 days after the order was placed. This time between placing the replenishment order and the arrival of the goods at the retail store is termed lead-time. The lead-time distribution is as follows:

Lead time (days)	Relative frequency
6	0.05
7	0.25
8	0.30
9	0.22
10	0.18

Demand that arises when the retailers is out of stock is lost, since customers whose demand cannot be satisfied immediately go elsewhere to carry out their business. Initially there are 100 units in stock. The retailer wants to know how his experience of stocking the item will vary, depending on where he sets the re-order point.

Build a model for the retailer's situation. Design the model to obtain a table over ‘lost daily sales’ and ‘number of units carried in inventory’. Run the model to estimate these two distributions, when the re-order quantity is 100 and the reorder point is 80. Shut off the model after a simulation of 1000 days. Ignore any weekend problems, by assuming that the retailer does business seven days a week and that a replenishment order continues to proceed also on Saturdays and Sundays. (47)

55. A bus is scheduled to arrive at a bus stop every 30 minutes, but it can be up to 3 minutes late. All delays between 0 and 3 minutes are equally likely. If a bus is late or not is completely independent of whether or not the preceding bus was late. A delay will not affect succeeding buses either. People arrive at the bus stop according to the (negative) exponential distribution with an average of 12 persons each 30 minutes. The bus, which has a capacity of 50 passengers, will have between 20 and 50 passengers on board when it arrives, with every number between 20 and 50 equally likely.

Between 3 and 7 persons (with 3, 4, 5, 6 and 7 equally likely) get off at the bus stop through the only door for exit from the bus.

After this, as many as possible of the waiting passengers will get on the bus. Those who cannot get space on this bus will walk away and will not return to the bus stop. It takes a time of 4 $\pm$ 3 seconds for a passenger to get off and a time of 8 $\pm$ 4 seconds to get on. Boarding of the bus cannot start until everyone who wants to get off the bus has done so. The passengers get off, and get on, the bus, one at a time. A person, who arrives while the bus remains at the bus stop, can get on, provided there is space left. The bus leaves as soon as either every one waiting has boarded the bus or the bus is full.

Write a simulation program to model this. The program should provide information on the distribution of passenger waiting time, divided into intervals of five minutes, and how many of the waiting passengers did not get on the bus. Simulate 25 bus departures from this bus stop. (47)

56. Alter Model44, so that it produces statistics referring to the steady state conditions. First write one version of the program to determine when steady state conditions begin, by allowing a study of the changes in the length of waiting for new books to arrive, measured month by month (of 25 working days). Next write a version producing steady state statistics, simulating a year. (48)

57. Modify Model60, so that it, for one order quantity, produces a report in a matrix table for each quarter (every 62.5 working days) on direct sales, delayed sales, inventory costs, ordering costs and profits. For simplicity, we use the size of the storage at the end of the quarter as the average contents of the storage of the quarter. (49)

58. An oil storage depot distributes three grades of fuel: home heating oil, light industrial fuel oil, and diesel fuel for road vehicles. There is one pump for each grade of fuel, and the demand for each is the same. Orders for fuel oil vary between 3000 and 5000 gallons, in increments of 10 gallons, evenly distributed. The time required to fill fuel trucks is a function of the pumping rate (6, 5 and 7 minutes per 1000 gallons respectively), the order size, the number of vehicles in the depot (30 seconds extra per vehicle) and the setup time, which is a fixed time of two minutes. The depot can hold a maximum of twelve trucks. The mean arrival rate of trucks is 18 minutes, modified by the following data:

Frequency	0.20	0.40	0.25	0.15
Ratio to mean	0.45	1.0	1.5	2.0

Simulate the operation of the oil storage depot for 5 days to find the distribution of transit times of trucks and the total quantity of fuel sold each day.

59. A super market has three checkout counters at its northern exit and two checkout counters at its southern exit. Customers arrive every 0.7 minutes, following an exponential distribution. 45 per cent of them go to the southern exit. At each exit, the customers go to the checkout counter which is free and, if all are busy, to the one with the shortest waiting line. They here spend 2.6 – 3.4 minutes. The manager wants to determine how the longest of all the waiting lines at the five counters varies over the day. He wants a report on this every ten minutes during the eight hour day.

60. Modify Model44 by incorporating the possibility of influencing sales by changing the price, assuming that the number of students per year q asking for the book is a function of price, $q = ap^{-b}$, where p is the price, a is a scale factor and b the price elasticity.

Let *a* = 30000 and *b* = 1.5 (implying that sales go up by 1.5 percent, if price goes down by 1 percent). Simulate both for a price of 25 and a price of 30. All other factors are the same as in Model44.

61. Extend Model60 to allow for the simultaneous determination of the best combination of order quantity and replenishing level. We shall here limit ourselves to the simple case that each of these decision values can be 10, 15 or 20, i.e. we have a total of 9 combination cases. One way is to have the number of the case as the decision variable and two functions that relate the number of the case to the value of the order quantity and the replenishing level.

62. The InterArrival Times of cars approaching a gas station and the service times are distributed as follows:

Interarrival Time (in seconds)	Cumulative frequency	Service time (in seconds)	Cumulative frequency
0	0.00	100	0.00
100	0.25	200	0.06
200	0.48	300	0.21
300	0.69	400	0.48
400	0.81	500	0.77
500	0.90	600	0.93
600	1.00	700	1.00

A car stops for service only if the number of cars already waiting for service is less than or equal to the number of cars currently being served. Cars that do not stop go to another gas station. The gas station is open from 7 a.m. until 7 p.m. Cars arriving later than 7 p.m. are not allowed in, but cars already waiting at 7 p.m. are served before the attendants leave for the night. It is estimated that the profit per car served averages $2.00, excluding attendants' salaries and other fixed costs.

Attendants earn $6.50 per hour and are paid only for a 12-hour day, even if they stay beyond 7 p.m. Other fixed costs amount to $75 per day. The station's owner wants to determine how many attendants (1 - 4) he should hire to maximize his daily profit. Build a model of the operation of the station. Simulate with each configuration for five different days.

63. A certain machine uses a type of part that is subject to periodic failure. Whenever the part fails, the machine must be turned off. The failed part is then removed, a good spare part is installed, if available, or as soon as one becomes available, and the machine is turned on again. Failed parts can be repaired and reused. The lifetime of a part is normally distributed, with a mean of 350 hours and a standard deviation of 70 hours. It takes 4 hours to remove a failed part from the machine. The time required to install a replacement part is 6 hours. Repair time for a failed part is normally distributed, with mean of 8 and a standard deviation of 0.5 hours.

The machine operator is responsible for removing a failed part from the machine, and installing a replacement part in its place. There is one repairer who is responsible for repairing failed parts. The repairer's duties also include repair of items routed from another source. These other items arrive according to a negative exponential distribution with a mean arrival time of 9 hours. Their service-time requirement is 8 $\pm$ 4 hours. These other items have a higher repair priority than the failed parts used in this particular machine.

Build a model for this system, to estimate the fractional utilization of the machine as a function of the number of spare parts provided in the system. Study the system under the alternative assumptions that zero, one and two good spare parts are provided initially. Run each simulation for the equivalent of five years, assuming 40-hour work weeks.

64. We shall here modify Model07 to allow for reneging. Customers leave the waiting line if they have waited 40 minutes without being served. To handle this in aGPSS, a SPLIT block is used. With this block we divide each customer into two fictitious "half-persons"; the left half goes as the normal customer into the waiting line; the right half stays outside of this waiting line, waiting to renege after a certain time. When it is time for the left half to leave the waiting line and go into service, it checks if the right half has already reneged, in which case it just leaves the system.

If the customer instead gets served before reneging time, it sends a signal to the right half that it has left the waiting line. When it is time for the right part to renege, it checks, if the left half has already gone into service, and if this is true, it just leaves the system; else it signals to the left half that it has reneged and then leaves.

The question is how the two halves can signal to each other. Since several signals, all for different customers, might exist at the same time in the system, each customer must have its own signal. Hence we must have a parameter, e.g. p$cust that gets the values 1, 2, 3, etc. for the different customers.

In Model89 we used Q(p$bestq) to denote the length of the AD set for the different counters. Likewise, we can create savevalues using this kind of indirect addressing. X(p$cust) can be used to denote whether a customer has stopped waiting, i.e., has either reneged or gone into service. In that case X(p$cust) = 1. Otherwise, i.e. if the customer is still waiting, X(p$cust) keeps its initial value 0.

65. This application deals with a very simple case of simulation based costing, but it exemplifies some of the basic ideas why simulation based costing is important.

Stochastic variations as regards e.g. machine time and arrival rates of one product can also influence the cost of other products. We study a factory where two types of products, A products and B products, are made in the same machine. The A products arrive exactly every 10 seconds and require a time of exactly 5 seconds.

The B products arrive every 20 seconds on average and require on average 10 seconds of machine time. For the B products we study two cases. In the first case there is no uncertainty and, just like for the A products, the sampled times are the average times. In the second case we have uncertainty, in which case the sampled times follow the negative exponential distribution of the average times.

We can study the two cases by inputting either 1 for certainty or 0 for uncertainty. For each case we simulate for 30000 seconds and print the number of A and B products and the cost of each product. The cost per unit is equal to the average time used in the machine times the cost of one second of machine time, which in turn is calculated as 100000 divided by the total number of machine seconds used.

66. A regional airline flying Dash Q400 planes with a capacity of 75 seats is contemplating its policy as regards how many days in advance of the flight they shall require passengers to book to get a low air fare.

It sets up the policy 100 days in advance for a flight. It knows that the expected number of passengers wanting to fly on a specific day will increase dependent on the distance from this start date. The closer one comes to the flight on day 100, the more passengers will want to fly. The average time between two bookings is estimated to be 1-CL/200, where CL is the number of days since the simulation start 100 days before the flight. A great variation, day from day, is however likely, so that the sampled date between two bookings is estimated to follow the negative exponential distribution.

60 percent of the passengers want to fly economy class. They pay $75 per passenger. If no cheap flights are available, they turn to another airline. 3 percent of the economy passengers do not show up, but they cannot rebook. Of the business class passengers, each paying $ 150, 20 percent do not come to this flight, but can rebook to another flight and their payment is not included in the income of this flight. If there are more than 75 passengers showing up, the airline has to pay some passengers $300 to get them to give up their seats.

The airline's main decision is the last date of booking flights at the economy price, since one does not want to turn away business passengers. Calculate and print the profit as income minus total costs.

67. A production shop is comprised of six different groups of machines, as follows:

Group number	Machines in group	
	Kind	Number
1	Casting units	14
2	Lathes	5
3	Planers	4
4	Drill presses	8
5	Shapers	10
6	Polishing machines	4

Three different types of job, Type 1, Type 2 and Type 3, move through the production shop. Each job-type requires that operations be performed in a specified sequence, as shown below.

Job type	Machine sequence	Mean operation time (min.)
	Casting unit	125
1	Planer	35
	Lathe	20
	Polishing machine	60
	Shaper	105
2	Drill press	90
	Lathe	65
	Casting unit	235
	Shaper	250
3	Drill press	50
	Planer	30
	Polishing machine	25

The operation times are all exponentially distributed, with the mean time given above.

Jobs arrive at the shop in a Poisson stream at a mean rate of 50 jobs per 8-hour day. 24 percent of the jobs are of Type 1, 44 percent of Type 2 and the rest of Type 3.

Simulate the production shop for five 40 hour weeks. At the end of each week, print the average time in shop for each job type for this week.

68. The establishment of optimal equipment life is important in managerial economics, accounting (e.g. for determining the depreciation rate) and finance. A company does not want to spend too much on new expensive equipment, but, on the other hand, it does not want to have obsolete equipment leading to high costs.

One method for determining optimal equipment life is the **MAPI** method, which determines equipment life taking technical development into consideration. Its critical assumption is that new equipment leads to lower production costs than old equipment. A machine that is new this year leads to *inFact* lower costs than a machine that was new last year. Hence, a machine that is 5 years old has 5**inFact* higher costs than a machine that is new. Hence, for a machine with the age *cl,* the **extra** cost, *xcost,* this year, due to not having the latest machine model, is *cl*inFact.*

One next calculates the present value, *prVal,* of this extra cost as *xcost*pvFact, where pvFact* is the present value of getting one dollar *y* years from now. When we have an annual interest rate *irate*, we can, using discounting, define the present value factor *pvFact* as *1/(1+irate)^y*. For a machine that is kept *y* years, this value is next summed up for all years, going from 1 to *y.* To this total sum of present values *sumPrV* of the extra cost of having the equipment, we also add the initial investment payment *invest*.

On the basis of this total cost of having the machine for *y* years, we next calculate the **annual** cost *annui* by multiplying this *sumPrV* with the annuity factor *anFact* needed to distribute the cost over *y* years. This calculation of the **annual** cost makes it possible to compare machines with different lengths of life. The annuity factor *anFact* is 1 divided by the sum of present value factor *smPvFa,* i.e. *anFact* = 1/*smPvFa*, where *smPvFa,* in turn, is (*1-pvFact)/irate*.

In order to be able to investigate also the stochastic case when the inferiority factor is not fixed, we assume that it varies following the normal distribution. We hence set *infact* = *infaAv+fn$snorm*stDev*. In order to have the lowest cost as a maximum we use a minus sign for the annuity.

Using EXPERI, we can calculate the annual cost of having a machine differently many years, first for the case without stochastic variations, setting *stDev* to 0, then for the case of stochastic variations with *stDev* > 0. We input the interest rate in the usual terms, e.g. as 10 (percent), which would lead to *irate* = 0.1. We also input *invest*, *infaAv* and *stDev.*

69. Samuelson and Bazerman (1985) provided a bidding problem in the energy sector, for which aGPSS can provide a powerful and illustrative solution. The problem is as follows: Company A is considering acquiring company T by means of a tender offer in cash for 100 percent of T's shares. The value of T depends on the outcome of an oil exploration project that T is currently undertaking. If the exploration fails completely, T will be worth $0 per share, but in the case of a complete success, a share of T under current management will be worth $100. All values between $0 and $100 are equally likely.

Regardless of outcome, T will be worth 50 % more under A's management than under its current management. The price of the offer to T must be determined **before** A knows the outcome of the drilling, but T will know the outcome when deciding whether or not to accept A's offer. T is expected to accept any offer from A that is greater than the per share value of T under current management. What price should A offer?

A typical student view is that the expected value of T to its current owner is $ 50 and that is worth 50 % more to A and that A should bid in the interval of $ 50 to $ 75. Investigate what would be a suitable price, by building a model that calculates the expected value for A for a given price offer. Finally, use EXPERI to determine the optimal price bid to be made by A.

70. In Lesson 17 we studied the WAITIF block. This block can as A operand use only a server name and as B operand only a symbol (U, NU, E, NE, F or NF). In contrast to the IF block, it is not possible to use SNAs or constants as A and B operands. The reason for this is that the processor should make a specific check every time that it is possible that a change has occurred that could affect the WAITIF block. With a WAITIF block referring only to a specific server, the required amount of such checking is very small. If the WAITIF block instead could refer to any SNA, such checking would have to take occur in connection with **every** event. This could possibly lead to **extremely** long running times**.**

The lack of a specific block allowing for waiting with regard to any SNAs implies that one in some cases must add a **logical** SEIZE block to allow for waiting. We shall exemplify this by modifying Model44. Instead of every customer buying just one item, a customer buys 1, 2 or 3 items; 40 percent buy 1 item, 35 percent 2 items and 25 percent 3 items. This number is thus sampled from a random function and then stored in a parameter, e.g. called P$number. If this number of items is not in stock at arrival, the buyer will not buy any items directly, but go into a waiting line at the address NODIR until this number is available.

We here indicate that there is a shortage by putting a logical facility, e.g. called SHORT, into use. The customer will now wait if, and as long as, this facility SHORT is in use.

This shortage can possibly be removed by the arrival of goods in the reordering segment. A customer can then leave the WAITIF block and try to take P$number units out of the replenished stocks.

Since there might be many customers waiting in the WAITIF block, it might very well happen that stocks are depleted again before all waiting customers have been able to get their P$number units. Hence, when each customer leaves the WAITIF block, we check that there are at least P$number units in stock.

If this is not the case, there is a new shortage and the customer is sent back, e.g. to an address LACK, where he again puts the logical facility SHORT into use. The customer then goes back into the waiting line. In order to assure that he remains at the first place in the waiting line, he is assigned the highest possible priority, e.g. through a block LET Priority=250-CL, before going back to LACK. He will now have to wait at LACK for another delivery of goods.

The only change in the reordering segment compared to Model44 is that the facility SHORT is released on arrival of new products as a signal to the WAITIF block in the customer segment. Besides the new assumptions above, all other assumptions behind Model44 hold also for this modified model.

71. We shall look at a problem that in decision theory is sometimes called the “partner selection” problem. A young man will in his search for a partner meet *x* young ladies. He gives each lady a score. We assume that this score, with an average of 1000, follows a statistical distribution, e.g. the negative exponential, the normal or the uniform distribution. If he does not choose her, he cannot go back to her, but proceeds to the next lady. He knows, however, after meeting each lady what the highest score this far is.

One decision rule is now that he, after meeting *y* ladies, determines that the *critical* score is the highest score this far. He then selects the next lady that he gives a higher score than this critical value. If he after having met all *x* ladies, has not found anyone with a higher score than the critical value, he will remain single, leading to a score of 0. What is the optimal value of *y* for a given value of *x*? Write a program to try to answer this question for different values of *x*, choosing one of the three distributions mentioned above.

72. Some passengers arriving at the central railway station in Magdeburg, Germany, want to get home by cab. The driver is told the destination. For this, a code 1 – 8 is used. The frequency of travelers and the distance to go is as follows:

Destination	Percent	Time distance
1	20	17
2	11	13
3	5	25
4	7	17
5	15	16
6	5	18
7	16	13
8	21	17

The average distance in time is the time distance given above. The sampled time is then this average time ± 5 minutes. After bringing the passengers to their homes, the taxis all return to the station. The time back to the station is equivalent to the time from the station. Passengers arrive with an IAT of 2.8 ± 0.5 minutes. We simulate for 100 passengers arriving at their home. The simulation should be used to investigate how waiting times for passengers change if the number of cabs increases from ten to eleven to twelve.

73. We shall in this model use aGPSS for a type of very simple continuous simulation. Even if aGPSS is a tool for discrete events simulation, one can in certain cases, e.g. when we make forecasts over many years, but at most a century, and look at each year as a whole, use aGPSS for a basic type of continuous simulation.

Continuous simulation mainly deals with differential equations. We shall base our model on the following solution technique. We shall here use $y' = dy/dt$ for three different types of equation:

1. $y' = f(y) = ky$ for exponential growth

2. $y' = f(y) = k(y_{max} - y)$ for modified exponential growth

3. $y' = f(y) = ky(y_{max} - y)$ for the logistic curve, which can be seen as a combination of equations 1 and 2.

We next approximate the differential equation with a difference equation, i.e. we approximate dy/dt with $\Delta y/\Delta t$ to get $\Delta y/\Delta t = f(y)$. For the case that $\Delta t = 1$, e.g. when we step a whole year at a time, we set $\Delta y=f(y)$. With $y_{j+1} = y_j + f(y)$, and starting with a value y_0, we can next, step by step, calculate a value y_j for any year j.

We can next apply these ideas to a model that is meant to forecast the sales of a new equipment to monitor the heating of one-family houses in a country with initially 3 million house-holds. The number of households is growing annually at a certain rate, 100**HoldGr* percent

Today 40 percent of the households live in a one-family house. Most, but a maximum of 90 percent, of the remaining households will want to move from an apartment to such a house.

Every year 100**HousGr* percent of these remaining household move to their own house This development is thus in line with the modified exponential growth.

Now, a new type of computer system has appeared on the market that by a web-based app can monitor several functions, like heating, in the house. When we start the simulation, there are only 2000 such computers in use.

The growth of these computers follows the logistic curve, implying that, when the sales start, the usage increases almost exponentially, but after some years, as the market becomes more saturated, the development is more in line with the modified exponential growth. In function 3, $ky(y_{max} - y)$, we set k to 0.0005 and y_{max} to 80 percent of number of single homes.

We shall make three graphs, one for development of the number of households, one for the number of single homes and one for the ownership of these new computers.

We run the simulation for 50 years.

In order to see how each of these three functions work, we first input both of *HoldPr a*nd *HousPr* as 0 to look at the third graph showing the logistic curve. Next we input *HousPr* as 3, but keep the value of *HoldPr* as 0, and look at the second graph, showing the modified exponential curve, which illustrates how the growth in house ownership levels off. Finally, we input *HoldPr* and *HousPr* as 1, to see how ownership in computers develops also because of changes in the number of households and the number of one-family homes.

74. At a certain library, anyone wanting a book must present a checkout slip to a clerk working behind the checkout desk. This clerk then goes into the stacks to find the book and returns to the desk with it. Build a model of the checkout procedure under the following conditions:

1. People wanting to check out books arrive at the desk in a Poisson stream at a mean rate of 30 per hour.
2. Each person wants to check out just one book and the wanted book is always available.
3. The number of clerks working at the desk is a decision variable in the model.
4. Whenever a clerk becomes available, the clerk is willing to pick up checkout slips from as many as four people at a time, if that many are waiting for service.
5. The time required to pick up a checkout slip is negligible.
6. A one-way trip between the checkout desk and the stacks requires 1 $\pm$ 0.5 minutes.
7. The time to find one, two, three, or four books is normally distributed with mean values of 3, 6, 9 and 12 minutes, respectively, and with a standard deviation of 20 percent of the mean.
8. When a clerk returns from the stacks, the time required to complete the rest of the checkout procedure is 2 $\pm$ 1 minutes per person.

Estimate the distribution of time spent by each person at the checkout desk, the average number of slips that a clerk picks up and the utilization of the clerks. Gather this information for the alternatives of 3, 4, and 5 clerks. For each alternative, simulate until 100 persons have been completely served.

75. Extend the model of Exercise 63, which dealt with a machine that used a single part, which we now call the A part. We now allow the same machine to use also another type of part, the B part. Just like the A parts, the B parts are subject to periodic failure. Whenever the A or B part in use in the machine fails, the machine must be turned off. The failed part is then removed, a good spare is installed, if available, or as soon as it becomes available, and the machine is turned on again. Note that, when one part fails, the time until the other one fails changes. If, for example, initially the A part had 330 h. left and the B part 415 h. left, then when the A part fails at time 330, the B part will only have 85 h. left before failure.

Like the A parts, the B parts can be repaired and then reused. The lifetime of B parts is normally distributed, with a mean of 450 and a standard deviation of 90 hours. It takes 4 hours to remove a failed B part from the machine and 6 hours to install a replacement part. The time required to repair a B part follows the distribution below.

Repair time in hours	Probability
5	0
6	0.22
7	0.35
8	0.26
9	0.17

Except for the added feature of the B parts, the conditions under which the machine operates are identical to those in Exercise 63. Use the modified model to estimate the utilization of the machine as a function of the number of A and B spare parts in the system. Simulate the system for each combination in which 0, 1 or 2 spares of each type of parts are provided. Run each simulation for 5 years of 40 weeks each.

76. Up to four barbers work in a barber shop. Intervals between successive customers are exponential, with an average of one every 10 minutes during time 0 - 120 and 225 - 390 (minutes after opening) and one every 5 minutes during time 120 - 225. No customers are generated after 390, but customers already in the system are served until ten hours after opening time. Lunch hour is as follows: The first barber goes to lunch at time 60 for a lunch of 30 minutes. The other barbers can then go to lunch, also for 30 minutes, one after the other. A barber goes for lunch first when he is idle, e.g. when he has finished serving the customer.

There are 3 waiting chairs. If an entering customer cannot be served at once he will leave immediately with the following probabilities: 0.1 if 3 waiting chairs are empty; 0.3 if at least 2 waiting chairs are empty; 0.5 if one waiting chair is empty and 1 if no such chair is empty. After having waited 15 minutes, a customer leaves unserved with a probability of 0.5. Service time is normal with a mean of 20 minutes and a standard deviation of 4 minutes. Determine the number of customers served per day for 1, 2, 3 or 4 barbers.

77. Tankers arrive at a harbor every eight hours on the average, exponentially distributed. The first tanker arrives at time 0. They then discharge their cargo into shore tanks. When a shore tank is full, or nearly so, i.e. has less than 20 K (20,000) tons capacity free, its contents are automatically transferred to the refinery. While this transfer is taking place, a shore tank may not be filled by a tanker.

The time to set up a pump is 0.5 hours. The pumping rate is 1 K tons per hour and the discharge rate is 4 K tons per hour. Tanker loads are either 15, 20 or 25 K tons, all equally likely. All shore tanks have a capacity of 70 K tons.

Run the simulation for 1000 continuous hours with the five shore tanks. Initially, two shore tanks are empty and free; one is currently discharging and will be free after 8 hours, and the other two are currently being loaded and will be freed after 12 hours (with 45 K tons contents) and 3.5 hours (with 25 K tons contents) respectively.

78. A port in Africa is used to load tankers with crude oil for overwater shipment. The port has facilities for including as many as three tankers simultaneously. The tankers, which arrive at the port every 11 ± 7 hours, are of three different types. The relative frequency of the three types and their loading-times are as follows:

Type	Relative frequency	Load time (hr.)
1	0.25	18 ± 2
2	0.55	24 ± 3
3	0.20	36 ± 4

There is one tug at the port. Tankers of all types require the services of this tug to move into a berth, and later to move out of the berth. The area experiences frequent storms, and no berthing or deberthing can take place when a storm is in progress. When storms occur, they last 4 ± 2 hours. The time between the end of one storm and the start of the next one follows the exponential distribution with a mean of 48 hours. When a tug is available and no storm is in progress, berthing or de-berthing takes 1 hour.

A shipper is considering bidding on a contract to transport oil to the UK. He has determined that five tankers are needed for this task. They enter the port 2 days apart and then require 21 ± 3 hours to load oil at the port. After loading and de-berthing, they would travel to the UK, offload the oil, return to the port for reloading, etc. Their round-trip travel time, including offloading, would be 240 ± 24 hours.

Simulate the operation of the port if this plan goes through for one year. Measure the distribution of the in-port residence time of the proposed additional tankers, as well as of the three types of tankers that already use the port.

79. A small elevator has space for only one person and goes between two levels. People wanting to go up arrive at the rate of one every 10 minutes, following an exponential distribution, while people wanting to go down arrive only every 20 minutes (also following an exponential distribution), since many prefer to walk down the stairs. The elevator takes 2 minutes in each direction.

On the ground-level there is only a button for going up and on the top-level only a button for going down. If the elevator is on the top-level and someone on the ground-level is pressing the button for going up and there is no one waiting to go down, the elevator goes down empty.

Likewise, the elevator goes up empty, if someone on the top- level is pressing the go down button and the elevator is on the ground-level and no one wants to go up. In the morning the elevator is on the ground-level. Simulate a day of 20 hours to determine the utilization of the elevator.

80. A wholesaler has a depot, where goods are delivered by truck. The goods are brought to the customers by van. The depot opens at time 0 and closes at time 360 (time in minutes). After time 360, no additional vehicles (trucks or vans) are allowed into the depot, but the trucks and vans already in the depot finish their tasks. The time between two trucks or two vans arriving follows the exponential distribution with the average times of 30 minutes for trucks and 5 minutes for vans.

The serving of the trucks and vans takes place on the second floor in the depot. An elevator is used to get there. This elevator has space for one truck or one van. The time to transport a vehicle up or down varies between 1.5 and 2.5 minutes, equally distributed. This time includes driving the vehicle in and out of the elevator. If the elevator travels empty, it takes exactly 1 minute up or down.

The servicing of the trucks or vans takes place at special loading bays. There are at present two bays for trucks and three for vans. (Each kind of vehicle must use its own type of bay.) The servicing is carried out manually by a team of workers. A team can service either a truck or a van. The time per truck follows the normal distribution with an expected value of 30 and a standard deviation of 5 minutes. The time for a van is also normally distributed with an expected value of 10 and a standard deviation of 3 minutes. The teams work all day without a lunch break.

The following rules are used to control the traffic: The trucks have priority over vans, both when waiting at the elevator and at the bays. The vehicles wait in two lines at the elevator, one for trucks, one for vans, until there is a bay free for the particular type of vehicle. They are then brought upstairs to the bay. (This time is included in the 1.5 - 2.5 minutes for the elevator.) The vehicle might then have to wait until a team is free to give service. The elevator goes up or down empty if there is some vehicle waiting on the other floor.

Management has observed considerable waiting times, in particular for the vans. In order to reduce these times, management contemplates an increase in the number of teams from 3 to 4 and an increase in the number of van bays from 3 to 4. Simulate the effect of these proposals! (Hint: Do exercise 79 above first.)

81. A new type of IT-exhibition is planned to be located in the center of Stockholm, the capital of Sweden. The exhibition is to be open 7 days a week, but the number of visitors will depend on the day of the week. The attendance on week-days will only be 60 per cent of average attendance, while on Saturdays and Sundays expected attendance is 200 percent of average attendance. The average attendance will be dependent on the entrance fee. Only four different entrance fees are contemplated: 20 SEK, 40 SEK, 50 SEK and 100 SEK. For these fees the average number of visitors per day is expected to be as follows: 20 SEK: 370; 40 SEK: 330; 50 SEK: 300 and 100 SEK: 230 visitors.

The main entrance, towards a busy street, will be open each day between 10.00 a.m. and 4.00 p.m. During this time, visitors are expected to arrive in a random fashion with InterArrival Times distributed according to the negative exponential distribution. At 4.00 p.m. this entrance door will be locked, but those already inside will allowed to stay until they are content and all attractions will run until the last visitor has left the exhibition.

In case there are already 120 persons in the real exhibition area, no new visitors will be allowed to enter this area through an inner entrance, where visitors pay the entrance fee. They can then wait e.g. in a cafeteria inside of the entrance to the street, but outside of the actual exhibition area.

A visitor can either follow a guided tour or walk on his/her own. 25 per cent of the visitors are expected to go on their own, regardless of when a guided tour begins. Each guided tour runs with exactly 10 visitors. A guided tour will normally start first when 10 visitors have gathered for such a tour. There will be only one guided tour at a time.

In case there are already 10 persons waiting for a guided tour to start, other newly arrived visitors, who originally had wanted to follow a guided tour, will go on their own.

The guided tour starts with a walk through the Hall of Introduction, which takes exactly 10 minutes. For those who are on their own this visit to the Hall of Introduction will take between 3 and 11 minutes, with all times equally likely.

Visitors next come to a hall with demonstration computers. There are five such computers. Each visitor uses one such computer and runs through a multimedia program. Those who follow the guided tour get advice from the guide and work seriously with the program. 95 per cent of them are expected to spend between 6 and 14 minutes using this computer. For those who walk on their own the corresponding time is assumed to be between 4 and 8 minutes. All these times on the computers are assumed to be normally distributed.

If some computers are being used, while others are free, a newly arrived visitor will self-evidently go to one which is free. In case all computers are being used, a visitor will go the computer that has the shortest waiting line in front of it. The persons in a guided tour will in each waiting line always be allowed to go ahead of the persons who walk on their own. They are, however, not permitted to interrupt someone who has already started running a program.

After all members of the guided tour are finished with the multimedia program, the guided tour is over and a new one can start. The main tour is at this stage also over for the persons going on their own.

Before leaving the exhibition visitors will, however, spend between 2 and 6 minutes (with all times uniformly distributed) browsing in an IT-shop. At the end of this time 60 percent of the visitors will buy a heavily discounted computer diskette for 100 SEK.

There are two types of program diskettes, one for young people and one for adults. 40 percent of the diskette buyers will choose the youth diskette. It is not certain that there is a ready-made diskette available when a customer asks for it. One person works full time copying diskettes and a customer will have to wait until there is a copy of the desired diskette available. After this the customer leaves the exhibition.

The person producing the diskettes starts with this work at 10.00 a.m. and follows the principle that he will at each instance produce the diskette of which there at the moment is the smallest stock. In the case of equally large stocks, he will produce the adult diskette. It takes him between 2 and 4 minutes (uniformly distributed) to produce a diskette, independently of the type. When he has finished with one diskette he starts to produce the next one. He continues in this way until the last visitor has left the exhibition. Diskettes remaining at the end of the day are sent to another shop. Hence, there are no diskettes of any type at the start of the day.

The planners of the exhibition wish to simulate one day at this exhibition. They want to have a program which from the keyboard can read the day of the week (e.g. with 1 for Monday and 7 for Sunday), as well as the entrance fee. They want the program to provide information on the average time spent by the visitors at the exhibition, counted from the time they enter the inner entrance, and the total revenues from entrance fees and sales in the IT-shop.

82. A corporation faces the following demand on the market as regards expected annual sales of a certain product at different prices of the product:

Price in $ 1000	Annual sales in units
10	310
20	110
30	60
40	40
50	30

This sales quantity per year determines the average number of days between each order. The real times between each order will, however, follow the (negative) exponential distribution.

30 per cent of the customers belong to a category that receives a discount of 20 percent of the price. In case the demanded product is in stock it will be delivered. Otherwise, the customer will wait for delivery, but only if the corporation can promise a delivery within a month. The corporation will give this promise only if the planned monthly production is at least as large as the number of orders that at this moment are waiting for deliveries. After the product has been delivered the customers will on average pay after 45 days. Around 1/3 pay within a month, but around 20 per cent pay later than after 2 months and around 5 per cent pay later than after 3 months.

The products are made in a special machine. The corporation decides on a fixed monthly production in units and starts the production of a unit in a steady stream, so that the total number of production starts corresponds to the total planned production of the month.

At the start of the production of one unit, costs of $ 4,000 are incurred. The costs also lead to an immediate corresponding decrease in cash position. The time needed for the production of one unit follows the normal distribution and is such that in 95 percent of all cases the time will be between 2 and 3 hours. After this, the products are put into inventory. The person handling the machine receives a total of $ 4,000 per month at the end of each month.

A problem can arise as regards production due to the fact that the machine, now fairly old, will break down on average once a month, i.e. once every 200 hours. The actual time will, however, follow the exponential distribution. At machine break-down the production of the unit is stopped. The production of the unit is restarted when the machine has been repaired. The time to repair the machine is between 2 and 4 hours (uniformly distributed). The firm doing the repair charges $1000 per hour.

At the start of the simulation there are 5 finished units of the product in stock and $ 100,000 in cash. The production is run in one shift five days a week. For the sake of simplicity we assume that a month consists of 200 work hours.

Write a program to test various monthly production plans and to provide a report on how this will affect the cash position and the total contribution of this product to the profits of the corporation.

Assume that there are no other costs than repair costs as regards the machine. The cash position should be followed in a graph, covering all changes, while the report on the total contribution should come at the end of the year for which the simulation is to be done. The program should have both price and monthly production read from the keyboard.

83. We study a machine shop where work is done on parts that are purchased for $ 15,000 per batch of 10 units. The work is done in a machine of which one can have several working in parallel. The work is done in the following steps, in which one worker follows the product the whole time:

(1) The worker carries out process 1 in a machine that is idle. The machine is then used according to the following time distribution:

Minutes	Percent
60	5
70	7
80	36
90	35
100	13
110	4

(2) The worker moves the product with the aid of a crane, as soon as the crane is free, to a chemical bath and then back again to the machine. This will in 95 per cent of the cases take between 10 and 20 minutes, with all times following the normal distribution.

(3) The worker next starts carrying out process 2 in a machine, provided it is idle. The machine is then used according to the following distribution:

Minutes	Percent
80	2
90	7
100	28
110	33
120	24
130	6

(4) The worker moves the product with the aid of a crane, as soon as the crane is free, to a container. This takes between 25 and 35 minutes, with the times uniformly distributed.

After step 4, the product is ready and the worker can go back and start with step 1 again. In case step 2 and step 4 compete for the crane, step 2 shall have priority. When the crane has taken care of a product, the machine is free to be used for another product, handled by another worker.

At the end of the production process there is a container, which can hold 10 units of the product. Each finished product hence increases the number of products in the container by 1. When there are 10 products in the container, the container is brought over to inventories of finished goods, ready for sales, and one starts to fill up another container. Orders are for full containers with a batch of 10 units. Demand, i.e. sales per **month** (of 30 days), for these containers depend on price, according to a continuous demand curve, for which we give some point values below:

Price of unit in $ 1000	Monthly sales in units
10	410
20	140
30	80
40	60
50	40

The time between each order follows, however, the negative exponential distribution. If there is not any container in stock when an order arrives, 60 percent of the customers turn to a competitor, while the remaining 40 percent wait, in a FIFO-order, until there is a container.

Each worker costs $ 3,500 per month in wages + social costs, while the machines are rented from another company within the company group for $ 25,000 per month per machine. Each crane is rented for $ 45,000 per month. The company runs continuously, 24 hours a day, 30 days a month, in 4 shifts. The moment one worker leaves his shift another worker continues the activity in which the first worker was just engaged.

The company wishes to use the simulation model for determining a suitable number of workers per shift, machines and cranes. Management wants to simulate for one month (of 30 days), at the start of which there are 3 containers of finished goods in stock. Management also wants a simple report on the monthly profits of this activity for a certain combination of price, workers, machines and cranes.

84. We simulate the activities of a new computer company. We first simulate the production of each single unit and estimate the time between the start of the production of each such unit as the number of days per year, 360, divided by the annual rate of production, *qprod.* The value of *qprod* is to be read from the keyboard, e.g. as 50. Then we increase the total number of units produced this far, *totpro,* by one unit and move it into the inventory.

We next calculate the cost of producing this unit. This cost will, thanks to learning, decrease exponentially with a factor *learn* (set to 0.001) and the total number of produced units, so that the actual cost per unit *accost* is *cost0∗exp(-totpro∗learn*). *cost0* is a constant, set to 20, representing production cost per unit, when no unit has yet been produced. *exp* represents the exponential function FN$EXP. We next increase total costs and decrease cash by unit cost *acost* Cash is at simulation start set to 4000.

We check that cash does not become negative. In such a case the simulation run is interrupted.

Sales activity. There are two types of sales: New sales, i.e. sales to first time customers, and repeated sales. As regards **new sales** we distinguish between two types of customer: innovators and imitators.

For **imitators** we define annual demand *imisal = imimar∗demand*, where *imimar* in turn is *scale∗newsal∗(matur-newsal). newsal* is the accumulated number of computers sold up to now to first time customers, *matur* is the saturation level and *scale* is a constant (set to 0.0001).

The saturation level *matur* is allowed to grow from an initial value *satur* (set to 200) over time by a factor *dgrate*, which is the growth rate per day (set to 0.00001), i.e. *matur=satur∗exp(t∗dgrate),* where *t* is time in days.

demand is a linear demand function such that annual sales would be 100 units if price was 0 and 0 if price was 100. Sales are assumed to vary continuously between these two values. Price is read from the keyboard at the start of the simulation. This price is not changed during the simulation run. 50 might be a suitable value on price.

For **innovators** we assume the following demand equation:
inosal = inomar∗demand, with *inomar = scal1∗(matur-newsal).*
scal1 is a constant set to 0.002.

We start the simulation with a low value of accumulated sales to new customers, 10. This value 10 is also given to accumulated new sales to all types of customer, *totsal*.

Next we generate, in one statement, each individual sale to an imitator and, in another statement, each individual sale to an innovator. For each type of customer, we determine the average time between two orders as the number of days per year divided by the annual sales, *imisal* or *inosal*, to this type of customer. The actual sampled time between two orders is next determined by multiplying this average time with a value sampled from negative exponential distribution.

As regards potential new sales, we first check that accumulated sales to new customers, *newsal*, do not exceed the saturation level, *matur*. In such a case we terminate the transaction immediately.

As regards **repeated sales** we assume that 80 percent of all buyers wish to make a repeated purchase, by going to that part of the program that deals with repeated purchases. The remaining 20 per cent leave the system. A repeated purchase will occur after a certain time, which is stochastic. Nobody buys a new computer within two years = 720 days, but the time after this until a new computer is demanded will follow an Erlang distribution with three drawings from the negative exponential distribution and with an average of 600 days.

In this way we simulate the arrivals of orders, both regarding new sales and repeated sales. These orders will, however, not always lead to an immediate sale. In case the firm does not have any computer in stock, the customer must wait until a newly produced computer comes into stock. The firm will then first take care of the order from the customer, who has waited the longest. Hereby the total accumulated sales, *totsal*, is increased by one unit and, in the case the sale was to a first time customer, the accumulated sales to such customers, *newsal*, is also increased by one unit. The sale has an immediate positive effect on both cash and profits.

In connection with the sale, the sold unit is taken out of stock. In case stocks thereby become 0, the firm is not producing enough. The firm then increases the annual production rate *qprod* by a factor *beta,* set to 0.01. If, on the other hand, the number of units in stock on this occasion is still larger than the desired level of a buffer of 10 units, the firm decreases the production *qprod* by multiplying it by 1-*beta*

The program shall produce a graph of the development of cash over time as well as a graph over total accumulated sales, *totsal*, and accumulated sales to new customers, *newsal*. The program shall also at the end of each year provide information about the revenue, costs and profits of the year.

Both cash and profits shall be increased at the end of the year, due to the firm obtaining a payment of interest, calculated as 5 percent of the lowest value of cash during the year.

Simulate for a total of 5 years.

85. We study a market for identical products, e.g. wheat. There are two producers. Demand can be described by a linear demand function $p=a-bQ$, where $Q=q_1+q_2$. Both producers have the same constant variable unit cost c.

Firm *i*'s profit $v_i =q_i(p-c)=q_i(a-bQ-c)=q_ib((a-c)/b-Q)=q_ib(A-Q)$, for $A=(a-c)/b$. We write this on a new scale as $V_i=v_i/b=q_i(A-Q)$. Hence $V_1= q_1 (A- q_1- q_2)= A q_1-q_1q_1-q_1 q_2$ and $V_2 = A q_1-q_2 q_1-q_2 q_2$.

The non-cooperative solution, found already by Cournot, is obtained when each firm regards the competitor's quantity as given. The optimal quantities are then determined by

$V_1' (q_1) = A - 2q_1 - q_{2.} = 0$ and $V_2' (q_2) = A - q_1 - 2q_{2.} = 0$
$\rightarrow \quad 2q_1 + q_{2.} = A = q_1 + 2q_2 \rightarrow q_1 = q_2$
$\rightarrow A - 3q_1 = 0 \rightarrow q_1 = q_2 = A/3.$

We now want to simulate a market, where the two firms know neither demand nor costs, but only their own profits and own quantities offered in each period. In experiments, one has found that, under certain conditions, the Cournot solution will be obtained after some time also under these assumptions of no knowledge about demand and costs. The question is under what general conditions this can occur. In order to investigate this, a simulation model is necessary, since one in contrast to costly experiments can afford to test a great number of conditions by inexpensive simulations.

For each of the two firms we first input two starting quantities *oneOld*, *oneNew*, *twoOld* and *twoNew*. We also input the intercept of the linear demand function *aVal* and a change factor *chg*. We set the initial value of *size* to 1

A first single event is generated at time 0. This puts the starting quantities of the two firms, *oneOld* and *oneNew,* into a graph. Next we calculate the initial changes in quantity, *qch1* and *qch2,* where e.g. *qCh1 = oneOld – OneNew*. Another single event is then generated, at time 1, which puts *oneNew* into the graph at time 1.

Firm 1 next makes its moves in periods 2, 4, 6, etc., while firm 2, makes its moves in periods 3, 5, 7, etc. The parties will hence take turns making their moves. This sequential quantity setting is fundamental for the model, since it ensures that each party can see the effect of one's own latest move.

For firm 1's periods we start by calculating firm 1's old profit *oldP1* based on the most recent quantities, *oneOld* and *twoOld*. We next calculate the new quantity for firm 1, *oneNew*, by adding *qch1*, to *oneOld*. On the basis of *oneNew* and *twoOld* we next calculate the profit of firm 1 in this period. If this new profit is higher than the previous profit, we let the new change for firm 1, *qch1*, be in the same direction as the old change, namely as *(oneNew-oneOld)*size*. If the new profit is lower than the previous profit, we let the next change be in the opposite direction of the old change, namely as *(oneOld-oneNew)*size*. In case of equal, i.e. unchanged profits, we go with 50 percent probability to repeat the change and with 50 percent probability reverse the change.

Before we move into the next period, we give the value of *oneNew* to *oneOld*. We also update the value of *size* by multiplying it with the change factor *chg*, e.g. 0.995, so that *size* constantly decreases.

For firm 2's decisions in periods 3, 5, 7, etc., we have a similar segment to determine a sequence of values on *qCh2, twoNew* and *twoOld*.

Build a model for this Cournot process. The model should produce a graph, by inputting every new quantity for each party when it has been computed.

When a user runs this model, aGPSS will first ask for the values of *oneOld, oneNew, twoOld, twoNew, aval* and *chg*. Let us exemplify with the values 1, 2, 20, 19, 24 and 0.995. In this case, with the initial values of q_1 and q_2 fairly far apart, we will after around 40 periods get a convergence on the quantity 8. In this case with *aval* = A = 24, this is equivalent to the theoretical Cournot equilibrium of 24/3 = 8. If we input other values, convergence might not be so fast and in several cases there will not be any convergence.

86. The issue is the administration of secretarial work in a company. At present, the president has a secretary, called PSEC, who does tasks only for him and no other jobs. The secretarial tasks of the other employees are handled by three secretaries, who work in a secretarial pool. Hence all these general tasks go to this pool and are handled by the first secretary available in the pool.

The present situation has led to some problems. While there is often a long waiting line for the general tasks at the secretarial pool, the utilization of the president's secretary is not very high (less than 50 percent). A consultant has therefore suggested an arrangement, whereby also PSEC can to some extent be connected to the pool to do general tasks, when idle. Furthermore, the president should be able to use the pool, if PSEC is busy.

The president is, however, afraid of getting worse service than earlier. The consultant therefore outlines the following specific proposal: The president's tasks would first of all go to PSEC. If she is busy (with whatever job), the task would go to the pool. If the pool is also fully occupied, the task would go back to PSEC. She would in this case interrupt her present work, if this is a general task and instead do the president's task. She would resume work on the interrupted general task first when the president's task is finished.

A general task would first of all go to the pool. If the pool is completely occupied, the task goes to PSEC. If she is busy, the work goes back to the pool to go into a waiting line there.

The question is now how service would change, if this proposal is implemented. Therefore the consultant wants to do a simulation study, comparing the present system with the proposed system.

The inter-arrival times for the president's tasks vary between 20 and 50 minutes, while the IATs for the general tasks vary between 6 and 18 minutes. The president's tasks take between 15 and 25 minutes, when done by PSEC, and between 16 and 30 minutes, when done by someone in the pool, since the secretaries in the pool are less experienced. The general tasks take between 20 and 40 minutes when done in the pool and between 17 and 33 minutes when done by PSEC. All times follow the uniform distribution.

Simulate both the present system and the proposed system for a day of 400 minutes with 50 runs to see which system is best with regard to the time needed to finish the president’s jobs.

87. The simulation concerns a company that buys a product from a manufacturer and then resells it on the market. Demand for the product is dependent on both price and product quality. Demand is also a function of earlier sales, since some customers, after varying times, return to buy an additional unit of the product. Demand is characterized by a continuous function for first time buyers, exemplified by the following data:

Price	Annual demand in units
10	60
20	40
30	25
40	20
50	15

The sales conditions are similar to those of Model62. The part of the segment that deals with the customers who return for repeat purchases is the same, also with the same relationship between unit cost and the probability of no repeat sales.

If there are no units left in inventory, the customer goes to a competitor and the order is lost. The number of lost orders is to be recorded.

The financial consequences are, however, different from those of Model62. This refers to the blocks after the SPLIT block. First the customer decreases the contents of the storage with one unit. Then we also add up sales and costs of goods. These costs are, however, not connected with immediate payments. The payments (= cash outflow) for purchased goods are now to be handled in the inventory segment (see below).

Sales first lead to an increase in accounts receivable. The time after which customers pay back follows the Erlang distribution with an average of 45 days and n=3. The first 30 days are interest free. We hence calculate the payment delay time, then subtract 30 from this number of days. If this subtraction leads to a negative number, we set the value for the time of interest payment to 0.

Assuming that the daily interest that we charge our customers is 24/36000, i.e. 24 percent annually, we next calculate the interest a customer has to pay when paying for the product. This interest cost of a particular purchase is added to net interest. At the same time the total cumulative cash out-flows increase by this interest payment. Finally, accounts receivable decrease and cumulative cash inflows increase by the payment for the product.

After inputting price, unit cost and cash, we input replenishing level and order quantity, i.e. when to reorder and how much to reorder each time. The inventory segment is similar to that of Model44, but for each replenishment purchase we calculate the total purchase expenditure as the ordered quantity times the purchasing cost per unit. This value is the borrowed amount and we increase debts by this amount.

The company's pays the supplier after 50 days. The company pays an interest on the borrowed amount. The daily interest rate is 20/36000, i.e. 20 percent annually. The first 20 days are, however, interest free. These interest costs of a particular purchase are deducted from the net interest results. At the same time the total cumulative cash out-flows decrease by these interest costs. Finally as the borrowed amount is paid back, debts decrease and cumulative cash outflows increase by this amount.

The program should generate a report every 90th day, i.e., at the end of each of six quarters. In this report segment, the number of the quarter should first be increased by 1 and written. Net cash-flows is calculated as cash inflows – cash outflows and added to total cash. The value of inventories = the number of units in store times purchasing cost per unit. The month's closing balance of assets is cash + accounts receivable + inventories and profits are sales - total costs + net interests.

The results are to be presented in a result matrix table with a short explanation of the contents of the rows. The columns are determined by the number of the quarter and the rows, 1 - 13, represent, in order: sales revenues, total (purchasing) costs, net interests, profits, cash inflow, cash outflow, net cash flow, cash balance, accounts receivable, inventories, total assets, debts and equity = assets - debts. Note that sales, cash inflow, cash outflow, total costs and net interests are all quarterly, not cumulative, values.

We also want a graph over the development of cash over time.

88. We shall here study a problem that is of importance for the managing of many retail operations dealing with perishable products, like milk, fruit and meat. The manager of a local supermarket has a single, deep shelf for cottage cheese. A customer who comes to buy a container of cottage cheese will always grab the one at the front of this deep shelf. In order to avoid spoilage, and hence monetary loss or loss of reputation to the supermarket, how should the stock boy be instructed to place newly arriving containers of cottage cheese? Should he place them at the front of the shelf so that customers will grab them first, or should he place them at the back of the shelf, behind containers that have not yet been sold?

The shelf can hold 20 containers of cottage cheese. Every 3.5 days (Monday morning and Thursday afternoon) new cottage cheese containers arrive from the distributor. The distributor leaves just enough containers to fill the shelf (e.g., if there were five containers on the shelf when the distributor arrived, she would leave 15 new containers). Customer arrivals are exponentially distributed with an average of one day apart. A container is considered spoiled if it has been on the shelf for more than 10 days when it is sold.

We simulate one year (365 days) and provide for statistics on the number of containers that were sold in a spoiled state for the two different methods for stocking the shelf.

Hints: Use a SPLIT block to create enough containers to fill the shelf with cottage cheese containers, when both the copies and the original are considered. Use an ARRIVE block to start measuring the time a container spends on the shelf before being purchased by a customer and a DEPART block to conclude the measurement of this shelf time. Use SEIZE and RELEASE blocks to assure that only one container at most can be at a position in the front of the shelf at any given time.

Use a WAITIF block, so that the container in the front of the shelf must first wait if and as long as a customer is not in the store and another WATIIF block to have it wait, while a customer is actually picking the container from the front of the shelf. Use a QTABLE statement with only 2 classes and a class width of 10, so that any times greater than 10 minutes will be tallied as an overflow, representing cottage cheese containers sold in a spoiled state.

89. Customers arrive by car to shop at a supermarket. The parking lot has space for 650 parked cars. If a customer fails to find a parking space, the customer leaves immediately without shopping. A customer can walk to the supermarket from the parking lot in 60 seconds. Shoppers purchase between 5 and 100 items, uniformly distributed. A customer buying 10 items or less, will generally use a basket (70 provided). A customer buying more than 10 items will generally use a cart (650 provided).

Shopping time per customer depends on the number of items purchased (8 seconds per item). Customers select items and then join the shortest queue at one of 20 checkouts. Customers purchasing less than 10 items may choose the express checkout. Checkout time takes 2 seconds per item purchased, plus a time of 25, 30, or 35 seconds. This time depends on the method of payment (cash, check or credit card which are assumed equally likely or probable). After checking out, a customer walks to the car (60 seconds), loads goods and leaves the parking lot. If there are no carts or baskets available on arrival, customers wait until one is returned.

The arrival rate of customers is exponentially distributed, starting at 600 per hour for half an hour, then 900 per hour for one hour, then 450 per hour for another hour and finally 300 per hour thereafter.

Run the simulation for 3 hours and determine the transit time of customers, as well as the utilization of the parking lot, carts baskets and checkouts.

90. In a duopoly situation, of the so called Bertrand type, the two firms produce identical products. If one firm has a higher price than the other, it will not meet any primary demand, i.e., it can only sell if the other firm has not enough stocks. If it has a lower price *p*, it faces a demand corresponding to potential annual sales $q = ap^{-b}$, where $a = 30000$ and $b = 1.5$.

To model this, we first input initial values of some constants, like initial prices and costs, *price1*, *price2*, *cost1* and *cost2*. We also define the demand that a firm faces, if it charges a lower price, *lowrpr*, than the competitor. This is equivalent to $q = ap^{-b}$. We also input the two constants *prilow* and *priadd*, used for changing the prices based on stocks. We finally input *protim*, production time.

Since we name the firms 1 and 2 so that firm 1 is the firm with the initially lower price, we also set the temporary lower price, *lowrpr*, initially equal to *price1*.

We also define the capacities of the inventories of the firms, allowing them a very high capacity. At the start of the first week there are no inventories. As starting values we also set, for each firm, the sales of the preceding week (before the start), *salLa1* and *salLa2*, as the highest possible weekly sales.

The model should have two main segments. In the first one we generate a report and decision event each week of 7 days, starting at time 0. We here first print the number of the week and then the profits made this far, as well as the number of units in stock of each firm.

Next we check if firm 1 has no stocks. If so, the price has been too low and *price1* is increased by *priAdd.* If not, i.e. stocks are not empty, price has been too high and we decrease *price1* by *priLow* times the number of units in stock, so that price is lowered more if the stocks are large. The price may, however, not be lower than unit cost. In that case *price1* is set to *cost1*. The changes in price done by firm 2 are similar.

In this first segment we also deal with production. Each firm sets its production quantity equal to the amount it sold in the preceding period. After a potential production period of *proTim* days, the products are ready to be put into inventory to be available for sales. However, if a firm's stock is already larger than its planned production, then no production takes place this week. We also determine what is now the lower price of *price1* and *price2*, and set this as *lowrPr*.

In the other main segment we generate every single order from customers wanting to buy at the lower price. The average time between two such orders, measured in days, is 52*7 divided by the number units demanded annually. The actual time between two orders varies, however, stochastically according to the negative exponential distribution.

If firm 1 has the lower price, we go to the address *sltes1*, where we test if firm 1 has any stocks. If so, firm 1 can sell and we go to *sell1*. If firm 1 does not have any stocks, demand goes to firm 2, i. e. we go to *sell2*. If firm 2 does not have any stocks either, demand goes back to firm 1, where one waits until it has got stocks again.

If firm 2 has the lower price, we go to the address *sltes2*, where similar conditions apply. If the firms have equal prices, we proceed by random with 50 percent chance to *sltes2* and with 50 percent chance to the next block *sltes1*

At the address *sell1* we might first have to wait until there is some unit in firm 1's stocks. We then take one unit out of firm 1's stocks and increase the number of sold units this week by 1 as well as firm 1's revenues by the price of the product. We also increase firm 1's costs of goods sold by the unit cost of the product. At the address *sell2* the corresponding happens to firm 2.

Build a program that simulates this situation and produces a graph for the development of the prices of the two firms for the first year as well as a weekly report on the cumulative profits and on inventories.

We can exemplify with the following input values: *price1*=24, *price2*=26, *cost1*=12, *cost2*=13, *a*=30000, *b*=1.5, *priLow*=0.1, *priAdd*=0.5, and *proTim*=5.

91. Karl is writing a business plan for a company he would like to start to produce private hobby airplanes. It would import the parts from the USA, assemble the planes and then sell the planes to customers in Estonia. Karl would like to simulate the operations of the company for the first two years, each of 240 working days.

Karl will use his business plan to obtain € 200,000 initial capital (in cash) from an investor. In case more cash is necessary at some point for operations, the company can take a loan of up to € 0.5 million from the bank. At the end of each month interest must be paid, 1% of outstanding loan (12% annually). The principal is not repaid during the first two years; only the interest is paid.

In case the loan has reached the maximum amount and more cash is still necessary, bankruptcy is declared and the simulation stops. Bankruptcy is also declared if the equity (assets minus liabilities) on the quarterly balance sheet is negative.

The parts are shipped from overseas in ship containers, each containing up to a maximum of 5 sets. The price for delivering a single container is € 20 000 and each set of parts costs € 40 000.

Karl will consider making an order every three weeks (15 workdays), the first time on the first day. If at that moment there is no order pending and the stock of plane parts (including WIP) is less than or equal to some fixed reordering level, he will make an order.

Each order is made for a single container and Karl can choose any quantity of sets between 1 and 5. (Both the reordering level and the quantity of sets will be kept fixed throughout the simulation.) It takes between 24 and 30 working days for a container to arrive, each real value in the interval being equally likely. The invoice must be paid within 15 workdays after an order is received.

The planes will be assembled by mechanics working for the company. The plan is to rent a hangar (costs € 4000 per month), where the assembly of a single plane is possible at any given moment. It requires 2 to 5 mechanics for the assembly (monthly wage for each is € 3 000). Wages and rents are paid at the end of the month.

The average time (in working days) taken to assemble a single plane is given below. The assembly time is random, but in 95% of cases it is within ± 20% of the average.

Number of workers	Average assembly time
2	15
3	11
4	10
5	9

Once a plane is ready, it is sent to the airport and parked, ready for customers. The parking space can be considered unlimited, but the company has to pay a parking fee. The fee, to be paid at the end of each month (of 20 working days), is € 1000 for each parked plane at that moment.

The customer arrivals are modelled as a Poisson process, i.e., the inter-arrival times follow the exponential distribution. The expected number of customers during one year depends on the price of the planes according to the data below. (For values between the listed prices, use continuous interpolation.)

Price in € 1000	Expected nr. of customers
60	25
65	18
70	15
75	10

If there is a finished plane parked at the airport when a customer arrives, it is sold immediately; otherwise customers join a waiting list. Once a plane is sold, an invoice is written. Half of the customers transfer the money within 27 workdays, but 5% take 63 days or more. The average time taken is 30.

Use the Erlang distribution with shape factor 3 and corresponding mean to model this delay.

Sales are the only source of revenue. Variable costs are the ordering and shipping costs of parts, and fixed costs are the monthly payments for rent, wages and interest. Profit **=** Revenue – Costs**.** On the balance sheet, all stock of parts, WIP and ready planes are valued at shipping + purchasing costs per item. Stock, Accounts receivable and Cash constitute Assets, while Loan and Accounts payable constitute Liabilities. Equity = Assets – Liabilities.

Model the physical processes described above and also keep track of the underlying accounting and cashflows. The produced output should contain a plot of cash & loan, quarterly printouts of the profit & loss statement and the balance sheet. All accounts should be in thousands of €, without decimals.

You should experiment and decide on suitable values for:

- Reordering level for parts (0 - 3)
- Quantity of sets per order (1 - 5)
- Number of mechanics to hire (2 - 5)
- Price for planes (€ 60 000 - 75 000)

The chosen values shall remain constant throughout the simulation. The objective is to have a high expected profit, while keeping the probability of bankruptcy during the first two years low. At least one run should be without bankruptcy. Customer waiting times should also be reasonably short.

92. We can by simulation estimate what in Management Science is called the value of perfect information. This refers to a situation with stochastic variations. One can for such a situation first determine what decision would be made, if one, due to perfect information, in advance knew what event would take place.

One can next determine what decision would be made, if one is not able to forecast what event will take place, but only which decision would maximize **expected** profits. The value of perfect information is then the difference in profit between the case with perfect information and the case with uncertainty.

We illustrate this with a situation of a newspaper seller, who has to order a certain number of a special foreign newspaper for sales each day. The number of papers that can be sold varies between 25 and 28, with 10 percent probability of 25, 30 percent probability of 26, 50 percent probability of 27 and 10 percent probability of 28. At the end of the day the newspapers are worthless.

Each unit ordered leads to a cost of $ 8, while each unit sold leads to an income of $ 10. If one has ordered fewer units than those demanded, sales only amount to the number ordered.

We test for the ordering of 25, 26, 27 or 28 units. This can for all aGPSS versions be done by first running 400 times, the first 100 runs with the order quantity of 25, the next 100 runs with 26, the next 100 runs with 27 and finally runs 301 – 400 with 28. For each of these four batches of 100 runs, we add up the profits and calculate the average profit for each such batch.

At the end of the 400th run, we determine which value of the order, 25 - 28, has given the highest average profit. This value then becomes the value of “non-perfect information”.

For runs 401 – 500 we set the order quantity equal to the sampled demand value and calculate the average profit for the case when we in advance know what demand value will be sampled.

At the end of the 500th run we calculate the difference between this average value and the value of “non-perfect information” and print this value as the Value of Perfect information.

93. We study the decision on the manning of a team that shall develop a new software product. The product is expected to be of interest only during the next four years. The company believes that the product cannot be sold after this. We are hence only interested in the profits during the next four years.

The product can only be put on the market once the development is finished. The expected value of the finishing time varies with the number of developers according to the following estimate:

No. of developers	Average development time
1	1000
2	500
3	280
4	240
5	200
6	180
7	165
8	155
9	150

There is a daily cost per developer of $ 1600. If the software has not been developed within three years, development is stopped. The development cost is to be paid at the end of each month. Once the development is ready, sales start in the immediately following month. Average sales are expected to be 2000 units per month.

Actual sales vary according to a normal distribution with a standard deviation of 400 units. The price of the software system is $ 299. The unit cost of production, incl. packaging with manuals, is $ 50.

All monthly profits are discounted back to the present. The discounting rate is 18 percent, corresponding to 0.05 percent per day. We want to estimate the total expected discounted profit for different numbers of developers, what variations are likely and which number of developers seems most suitable.

94. In a restaurant there are different synchronization problems. First, the waiter/waitress waits until each guest in a group has chosen a meal. In order to serve the meals for the group together, the meals wait until the last one for the group is prepared. Then they wait until one of two waiters/waitresses are free. The table is normally cleaned when all guests have finished their meal.

A groups of customers arrive on average every 10 minutes, following a Poisson pattern. The number of persons in each group and their relative frequency is as follows:

1	0.2
2	0.24
3	0.1
4	0.2
5	0.1
6	0.16

There are six tables in the restaurant; four tables for four people and two for six people. Each group waits until there is a table free to accommodate all in the group. If there are already three groups waiting, the whole arriving group goes to another restaurant.

For the number of drinks in each group, the time to consume these drinks, the time of each group to choose and next to order the food, the time to eat the food, and finally the time to pay and the time spent after paying, you are free to decide on what the values (possibly following distributions or functions of other factors) that you find most suitable. The same refers to the time for the barman to prepare the drinks, the cook to prepare the food, the time of one of the two waiters to serve and finally to clean the table.

95. We revise Model100 by assuming that the city is large enough to have five service bays and five mechanics in its vehicle-maintenance garage. We let the number of vehicles coming in for scheduled maintenance each day be uniformly distributed between 12 and 18, instead of between 2 and 4.

We continue to let police cars arrive for unscheduled repair in a Poisson stream, but now at a rate of five cars every two days, instead of one every two days. We use the same preemptive queue discipline that was used in Model 100 and use the other assumptions made there, i.e. that the mechanics work their 8-hour day without taking breaks and that the garage is open 7 days a week. We simulate for 25 days.

96. This simulation deals with beer drinking. The program should start by asking the user: “Want to drink Swedish beer in 40 cl glasses? Y/N”, with hopefully *Y* as the answer.

It then asks for the average time between sips in minutes and next for the average size of a sip in cl. Finally it should ask for how much beer in cl. you have left in your glass, when you order a new glass.

Then the simulation starts. The real time between sips follows the negative exponential distribution of the average input time. We assume that the true sip varies according to the normal distribution with the input average and with a standard deviation of 10 percent. The new glass of beer comes 10 minutes after the order, but varying with the normal distribution with a standard deviation of 2 minutes.

The drinking activity is recorded with a log with the following types of output lines:

```
 At time       0.00 I order one beer
 At time     10.91 I drink      5.26  cl beer
..
 At time      46.02 I order one beer
..
 At time      59.06 my glass is empty
```

The simulation is ended after five beers with the output:

```
Five beers are enough. Carry me home!
```

97. This example deals with the evaluation of European stock options. It is very simple and meant to give a taste of the role of simulation in determining the value of these options, i.e. to show both the potentials and problems involved.
ytu
We deal with a call option where the buyer (owner) of the option can buy the stock at a pre-set price, the so-called exercise price, after one year, if she wants to. We have for the model chosen the following formula for estimating the value of the stock at the end of a period:

y*exp((r-s^2/2)*t+s*sqrt(t)*fn$snorm)

y is the value of the stock at the start of the period, r is the annual interest rate as a fraction, s is the annual volatility of the stock, also given as a fraction and *t* the time after which the new value is reached.

We shall in the formula above have *t* as a month and we can then in 12 steps calculate the value of the stock after one year. Next we calculate the difference between this value and the exercise price. The value of the option at the end of the year is then the maximum of this difference and 0, since the option owner will only use her option, if the difference is positive. The value at the start of the year is then the present value of this.

For running the model, we first study the case when the starting price is 100, the exercise price is also 100, the annual volatility is 25 and the interest rate is 5 percent. For the case that we have constant volatility, we can compare the value obtained from the simulation with that obtained from Black&Scholes formula calculators.

The interesting thing with simulation in this case is to obtain option values also for a case when the volatility changes during the year, a case not covered by the Black&Scholes formula. We shall here only study the simple case when volatility changes from an initial volatility by a monthly increase. We can allow this change in volatility to be stochastic, so that this change follows the normal distribution with the change being the average change in volatility times (1+fn$snorm**stddev*), where *stddev* has been read-in. We can e.g. study the case when the average monthly increase in volatility is 1 and its standard deviation is 0.2.

We run the simulation a great many times. Since, for example, 400 times is the maximum allowed for experiments for the extended student version of aGPSS, we write the model so that we in the same run can obtain results for more than 400 runs.

We hence need one segment that goes from month 1 to month 12 for each run and one segment that starts every 12 months and then sets the start values for each year, i.e. for the number of the month and the initial volatility. Finally we need a segment that stops the simulation after *m* months, where *m* is the number of runs *12.

At the end of each run, we add the present value of the option value in this run to the sum of such present values in the earlier runs. In the last run we divide this sum with the number of runs to obtain the expected value of the option.

The desired output is this expected value of the option. It is also of interest to have the possibility to show a graph of the values of the stock at the end of the year.

The model should allow for input of the starting price, the exercise price, the interest rate, the initial volatility, the monthly change in volatility, the standard deviation of this change and the number of runs.

98. Our next model is a simple version of the models that our students have built for a Swedish company selling corporate switchboards. The company originally used an analytic queue theory model, but early simulation models showed that the analytical models lead to errors, e.g. regarding waiting times and lost customers, of up to 20 percent.

The model deals with an insurance company buying a new switch board. This company shall in co-operation with the seller of the switch board, having the simulation model on his laptop, determine the number of in-going trunk lines for the switch board and the number of agents that shall handle customer calls.

The arrival of customer calls follows the Erlang distribution with a parameter n of 3 with an average IAT that depends on the time of the day. The average IATs in seconds during the day are estimated to be as follows:

9 – 11	30
11 – 13	50
13 – 14	20
14 – 16	25
16 – 18	30
After 18	30

The switch board opens at 9 a.m.. After 6 p.m., no new calls are admitted into the system, but the calls having arrived before that are kept in the system until 7 p.m., when the system is closed down.

The calls first come to the switch board. If all trunk lines are busy, the caller has the choice of either calling back later or calling another insurance company, implying a lost customer. 70 percent prefer to call back later. They will then wait 10 minutes on average, with a standard deviation of 2 minutes. They then call back, but if there is still no free agent, they call another insurance company, implying another lost customer.

Those calls that are admitted into the system might still have to wait for a free agent. The time needed by the system for finding a free agent is 20 seconds. If all agents are busy, the caller is put on hold until an agent is free.

The caller then spends a certain time with the agent. To model this time, we assume an average of 4 minutes.

In order to have this reflect the risk of some lengthy calls, we assume a basic time with an average of 3 minutes, following the normal distribution with a standard deviation of 30 seconds. To this basic time we add an average of 1 minute, following the negative exponential distribution. After this total time, the call is completed and the agent and the trunk line are free for other calls.

At the end of the simulation of one day, we want to calculate the costs of these operations for a certain number of agents and trunk lines. The total costs, which are to be printed out, contain three components:

1. The salaries to the agents, which is the number of agents times the daily cost of an agent.
2. The cost of the switch board, which is the number of trunk lines times the daily cost of each trunk line.
3. The cost of losing a customer, which is the number of lost customers times the cost of losing a customer, i.e. foregone profits.

As start values we input the daily cost of an agent (e.g. 100), the daily cost of a trunk line (e.g. 50) and the cost of a lost customer (e.g. 200).

99. We shall here make a simulation of the value of a new high tech company. We shall do this by making a forecast of various factors such as sales, profits, dividends, year by year, for a great many years up to a horizon year T.

In the first step, step **a**, we build a completely deterministic model, with no random effects. We set the value of the horizon year to 40

We shall start with the forecast of the development of the market for the main product of the company. As a first step, we forecast the **total** market for the product that the corporation is about to sell. We shall first forecast the total market in terms of a constant economy. We shall for the forecast of this total market assume that the development of the sales of the firm's product follows the familiar pattern of the logistic curve, used in Exercise 73.

As the first parameter we set the market saturation level y_{max}, to 100. The next parameter is a growth rate g_y, set to 0.003. The market growth Δy_t from year t to $t+1$ is $g_y(y_{max}-y_t)\ y_t$, where y_t represents the total (relative) market in period t. We can in this way, by specifying an initial value, y_1, of y in year 1, 10 (e.g. K units), calculate the total market for year after year.

We also allow the total demand for the firm's product to grow in proportion to the expansion of the total economy, e.g. with population growth, i.e. a growth of a fixed percent each year in an exponential fashion. We shall here forecast an index-like value for the whole economy, the GNP, with a value of 100 for the first year, and then increasing by a constant growth rate of 3 percent. The total market for the product is then be defined as the earlier forecasted ”economy-independent” market multiplied by value for the total economy.

Having defined the total market, our next step is to determine the sales of our specific corporation. This will depend on the development of the market share of the corporation, starting at 10 percent. We assume that the market share s will develop over time, eventually gradually reaching a maximum market share of 25 percent. Thus the change in the market share s_t from period t to year $t+1$ is $g_s(s_{max} - s_t)$, where g_s is set to 0.15. Multiplying the total market in each year with the market share, we get the total sales of the studied corporation.

The next step is to determine how the annual profit develops over time. We assume that profit is dependent on the profit margin. This percentage will also develop over time. We assume that the profit margin *m* develops gradually over time from its starting value m_1, set to 10, to gradually approach a final value, m_{max}, of 20. The change in the profit margin from year *t* to year *t+1* is $g_m(m_{max} - m_t)$, where g_m is a growth rate for the profit margin, of 2 percent.

We must also include a fixed cost, of 100, leading possibly to initial losses. For the sake of simplicity we assume in this simple model that this fixed cost is constant over time. Multiplying sales with the profit margin and deducting the fixed cost, we obtain the development of profits over time.

Having established the development of the annual profit of the corporation, we next forecast the development of dividends. We assume a fixed dividend ratio, of 50 percent, such that the annual dividend is this ratio times the profit, provided the annual profit is >0 . In the case of a loss, the dividend is 0. We shall next in our model calculate the sum of the present values of all dividends up to the horizon *T*. We assume a discount rate of 10 percent.

In order to take also the dividends after the horizon year *T* into account, we first assume that all dividends after *T* are the same as the dividend at year *T*. We can next calculate the lump sum value at year *T* of all dividends **after** the year *T* as the present value of all dividends from the horizon *T* to infinite time, i.e. equal to the dividend in year *T* divided by the discount rate. We add the present value of this lump sum to the total value of the corporation.

We also have to include the effect of the profits not distributed as dividends, i.e. for each year the profit minus the dividend. We assume that these amounts are invested into a special reinvestment account carrying an annual interest, different from the discount rate, of only 7 percent. We allow this reinvestment account to grow each year until the horizon year *T*. In this year we calculate the present value of this reinvestment account and add it to the total value of the corporation.

The value of the company is thus calculated as the sum of the present value of all dividends from year 1 to the horizon year *T* plus the present value of the lump sum in year *T* plus the present value of the investment account.

In step 1 we want to have nine graphs over the development of 1. the market as such, 2. GNP, 3. total sales on market, 4. the market share, 5. sales, 6. margin, 7. profits, 8. the three components building up the total value and finally 9. the company's value.

In step **b** we want to investigate the effect of some random variations. We shall assume three variations, following the normal distribution. The GNP growth rate can vary between 0.01 and 0.05, the market maximum value between 80 and 120 and the margin maximum value between 10 and 20, all with a 95 percent probability that the sampled value will lie in between the lower and upper values given above.

We shall in this case abstain from graphs but just investigate to what extent the size of the horizon year influences the total value of the corporation. Will there be a significant difference in this value, if the horizon lies 30 or 40 years away?

100. The car producer Ovlov has close cooperation with the French Napoleon company. The two companies are now in the process of planning their purchase of a new metallic paint. This new paint will protect better against corrosion.

It is purchased from three different suppliers, one in Düsseldorf, one in Esbjerg and one in Torino. This paint is used in the three car plants, in Antwerp, on Hisingen and in Paris.

The demand for this paint is dependent on when and where the cars are produced. One opens a barrel with paint and fills the painting machine when one has a whole batch of 100 cars ready for painting. Before this, one has to clean the spraying nozzles. On average one has to open a new barrel with paint every 7.5 hours on Hisingen, every 8 hours in Antwerp and every 9 hours in Paris, but the times between these barrel openings vary and seem to follow the negative exponential distribution.

In case that one does not have any barrels with this paint, one has to wait with painting the 100 cars until paint barrels arrive. The delay of a series of 100 cars costs 500 € (i.e. 5 € per car) for each hour of delay for all three production units.

When simulation starts, there are stocks of 20 barrels in Antwerp, 20 in Paris and 15 on Hisingen. One has a capacity of stocking 1000 barrels in Antwerp, 500 on Hisingen and 400 in Paris.

The production of paint is done in batches of 10. In Düsseldorf it takes on average 20 hours to produce a batch of 10 barrels, but time varies so to that it in 95 percent of all cases takes between 12 and 28 hours, seemingly according to the normal distribution. In Esbjerg and Torino conditions are similar, with variations between 24 and

40 hours, and 24 and 36 hours, respectively. The barrels are stored while waiting to be fetched. There is a capacity to store 1000 barrels (i.e. 100 batches of 10 barrels) in Düsseldorf, 500 in Torino and 400 in Esbjerg.

This type of metallic paint hat to be transported in special trucks. At present there are three such trucks; at simulation start one in Düsseldorf, one in Esbjerg and one in Torino. They leave (in this order) for that one of the three plants that has the longest waiting line of unpainted cars. The average travel time in hours for the trucks between the places is given in the following table:

	Antwerp	Hisingen	Paris
Düsseldorf	8	22	12
Esbjerg	17	14	20
Torino	18	28	16

The actual time needed for each truck will, however, vary, seemingly according to the normal distribution, so that 95 percent of the times will lie between 60 and 140 percent of the average time. There is, however, also a lower limit on the time needed, given in the following table:

	Antwerp	Hisingen	Paris
Düsseldorf	6	16	9
Esbjerg	13	9	14
Torino	12	20	11

All times shown above include both loading and unloading, which is not very time consuming. When a truck has arrived at the car production plant, one unloads the 10 barrels. The truck is then ready to go back to one of the three paint producers. It will go to the color producer that has the largest number of barrels in stock. If no producer has any stocks, it will go to Düsseldorf. The times for the return travels are the same as the times going to the car producers.

Since the trucks are expensive and the trucks are manned by specially trained persons, one calculates with a cost of € 1000 per hour of truck travelling time. When the trucks are only waiting, there is an hourly cost of € 700 per hour.

After the trucks have returned to the color plants, they load 10 barrels, provided 10 are in stock; else they wait until 10 have come into stock. They next again determine which car factory has the longest line of waiting unpainted cars and go there. In this way, the trucks go back and forth between the color and car plants.

We simulate for 100 days of 24 hours. Both the car plants and the color plants work in many shifts around the clock. We want to calculate and print the total costs for the trucks plus the cost of production having to wait.

We want to simulate both for the present case of 3 trucks and the case of one additional truck, starting in Düsseldorf. Make enough runs to determine if it is worthwhile getting an extra truck.